1. Suppose $n \geq 2$ and let $A_1, \ldots, A_m \subseteq \Omega$ all have size $n$. Suppose $m < 4^{n-1}$. Show that there is a coloring of $\Omega$ by 4 colors so that no edge is monochromatic.

2. Suppose $n \geq 4$ and let $A_1, \ldots, A_m \subseteq \Omega$ all have size $n$. Suppose $m < \frac{4^{n-1}}{p}$. Prove that there is a coloring of $\Omega$ by 4 colors so that in every edge all 4 colors are represented.

3. The expected number of isolated trees [just take this as a fact] on $k$ vertices in $G(n, p)$ is given by $f(n, k, p) := \binom{n}{k} k^{k-2} p^{k-1} (1-p)^B$ with $B = k(n-k) + \binom{k}{2} - k + 1$. Set $p = \frac{1}{n}$. Let $c$ be a positive constant. Find the asymptotics of $f(n, k, p)$ when $k \sim cn^{2/3}$. (*) Express the limit as $n \to \infty$ of the sum of $f(n, k, p)$ for $n^{2/3} \leq k < 2n^{2/3}$ as a definite integral and use a computer package to evaluate the integral numerically.

4. Consider Boolean expressions on atoms $x_1, \ldots, x_n$. By a $k$-clause $C$ we mean an expression of the form $y_{i_1} \lor \ldots \lor y_{i_k}$ where each $y_{i_j}$ is either $x_{i_j}$ or $\overline{x_{i_j}}$. Prove a theorem of the following form [you fill in the $m = m(k)$] by the probabilistic method: For any $m$ $k$-clauses $C_1, \ldots, C_m$ there is a truth assignment such that $C_1 \land \ldots \land C_m$ is satisfied.

Working with Paul Erdős was like taking a walk in the hills. Every time when I thought that we had achieved our goal and deserved a rest, Paul pointed to the top of another hill and off we would go.

– Fan Chung