HW 3, Part 2: Computer Problems

Objective: To explore different versions of the QR factorization and its application.

1. Use the standard Householder QR to solve the problem 3.2, Page 153. In Matlab, the function call for Householder QR is \([q,r]=\text{qr}(A)\). The so-called best values of the altitudes are simply the least square solution to the linear system. Omit the questions given in problem 3.2, instead, answer the following questions
   (i) Provide the matrix on which you apply QR factorization
   (ii) Print out the solution \(x(1:4)\) in short e format, as in \(3.1416e+00\) for \(\pi\), namely in 5 decimal digits
   (iii) What is the quantity your solution minimizes.

2. Given \(m, n\) as input, write two Matlab functions to construct the Hilbert matrix \(H \in \mathbb{R}^{m \times n}\) defined by \(H_{ij} = 1/(i+j-1)\). The first version \(\text{hilb1}(m,n)\) uses two embedded for-loops. The second version \(\text{hilb2}(m,n)\) is fully vectorized (Hint: Consider the matrix \(B=\text{transpose}(1:m)*\text{ones}(1,n)\), and what is \(1./B\)). Answer the following questions
   (i) Print out the matrix \(H\) in short e format for \(m = 8, n = 5\)
   (ii) Provide the Matlab script for your vectorized version
   (iii) Write and provide a script of no more than 12 lines to call the functions for the case \(m = 2000, n = 1600\). Use \(\text{cputime}\) to show the runtimes for the two functions.
   (iv) Provide the condition number of \(H\) for case \(m = 8, n = 5\) and the case \(m = 32, n = 16\).

3. Given \(A \in \mathbb{R}^{m \times n}\) as input, write a Matlab function \([Q,R]=\text{qrmgs}(A)\) to implement the modified Gram-Schmidt QR, see Page 132. You must have the inner for-loop vectorized
   (i) Provide vectorized Matlab script for the function
   (ii) Run it on the Hilbert matrix \(H\) with \(m = 8, n = 5\). Do not provide the result \(Q\) or \(R\), instead, calculate and show the two backward errors

\[
\|Q^TQ - I\|_2 = \text{norm}(\text{transpose}(Q)*Q - \text{eye}(n)), \quad (1)
\]
\[
\|QR - H\|_2 / \|H\|_2 = \text{norm}(Q*R - H) / \text{norm}(H) \quad (2)
\]
   (iii) Repeat (ii) for \(m = 32, n = 16\)
   (iv) Repeat (ii) and (iii) using the Householder QR