Sylvester-Gallai-like Theorems for Polygons in the Plane

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The Sylvester-Gallai Theorem says that given a finite collection of lines in the projective plane there must be a point where exactly two of the lines intersect - a so-called ordinary point. I consider the analogous problem in the context of simple polygons without holes. Starting with such a polygon $P$, consider an arbitrary finite set of lines through the interior of $P$, together with the lines determined by the edges of $P$. Now consider the vertices, edges and cells of the arrangement that lie on or inside $P$. I call this set an arrangement in $P$ and derive various Sylvester-Gallai-like Theorems for such arrangements, given various different types of polygons $P$. I will also consider a colored cousin of the Sylvester-Gallai Theorem, known as the Motzkin-Rabin Theorem, and provide analogs of this Theorem, again in the setting of polygons.

I will describe how these results connect back to a famous problem, which Ricky Pollack calls the Quantitative Sylvester Problem, - namely for $n$ not all coincident lines in the projective plane, how many ordinary points must there be?

For more information please visit the seminar website at:
http://www.math.nyu.edu/seminars/geometry_seminar.html.