Random matrices: Universality of the spectral distribution and the circular law

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Given a $n \times n$ complex matrix $A$, let $\mu_A(x, y)$ be the counting measure generated by the (complex) eigenvalues of $A$.

We consider the limiting distribution (both in probability and in the almost sure convergence sense) of the normalized ESD $\mu_{\frac{1}{\sqrt{n}} A_n}$ of a random matrix $A_n = (a_{ij})_{1 \leq i, j \leq n}$ where the random variables $a_{ij} - E(a_{ij})$ are iid copies of a fixed random variable $x$ with unit variance. We prove a universality principle for such ensembles, namely that the limit distribution in question is independent of the actual choice of the atom variable $x$. In particular, in order to compute this distribution, one can assume that $x$ is real or complex gaussian. As a related result, we show how laws for this ESD follow from laws for the singular value distribution of $\frac{1}{\sqrt{n}} A_n - zI$ for complex $z$.

As a corollary we establish the Circular Law conjecture (in both strong and weak forms), that asserts that $\mu_{\frac{1}{\sqrt{n}} A_n}$ converges to the uniform measure on the unit disk when the $a_{ij}$ have zero mean. (In particular, this strengthens the result I discussed in a colloquium at NYU in November 2007)

The proof uses tools from additive combinatorics, probability and high dimensional geometry.

(The talk is based on a recent paper "Random matrices: Universality of ESDs and the circular law", by T. Tao and V. Vu, with an appendix by M. Krishnapur.)

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