Geometric Properties of Multivariate Bernstein Basis and Certificates of Positivity

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Abstract

The Bernstein basis on a simplex of polynomials of degree less than \( d \) can be easily defined. It is well known that in the univariate case, the Bernstein coefficients of a polynomial define a control line, so called because the shape of the graph of the polynomial can be easily controlled using the Bernstein coefficients. In the multivariate case, the definition of the control polytope cannot be given immediately from the Bernstein coefficients. In order to define the control polytope and study its distance to the graph of the polynomial, it is convenient to introduce a combinatorial construction: the standard triangulation of a simplex, and to characterize combinatorially the convex piecewise linear functions based on this triangulation. Two ways of making the approximation between the control polytope and the graph closer are considered: by elevation of the degree, or by subdivision. Short and adaptive certificates making visible that a polynomial is positive on a simplex follow from the subdivision method.