

Efficient Discovery of Heterogeneous Treatment Effects via Anomalous Pattern Detection

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UNIVERSITY OF MINNESOTA
Driven to DiscoverSM

Agenda

- Introduction and Motivation
- Machine Learning's Contributions (and Limits)
- How Anomaly Detection Can Help
- Treatment Effect Subset Scan
 - Algorithm
 - Statistical Properties
- Results
- Conclusions











Treatment Effect Heterogeneity

Control Group Treatment Group

	Control Group	Treatment Group
Age = Young	YC YC YC	YT YT YT
Age = Mid	YC YC YC	YT YT YT
Age = Old	YC YC YC	YT YT YT

Treatment Effect Heterogeneity

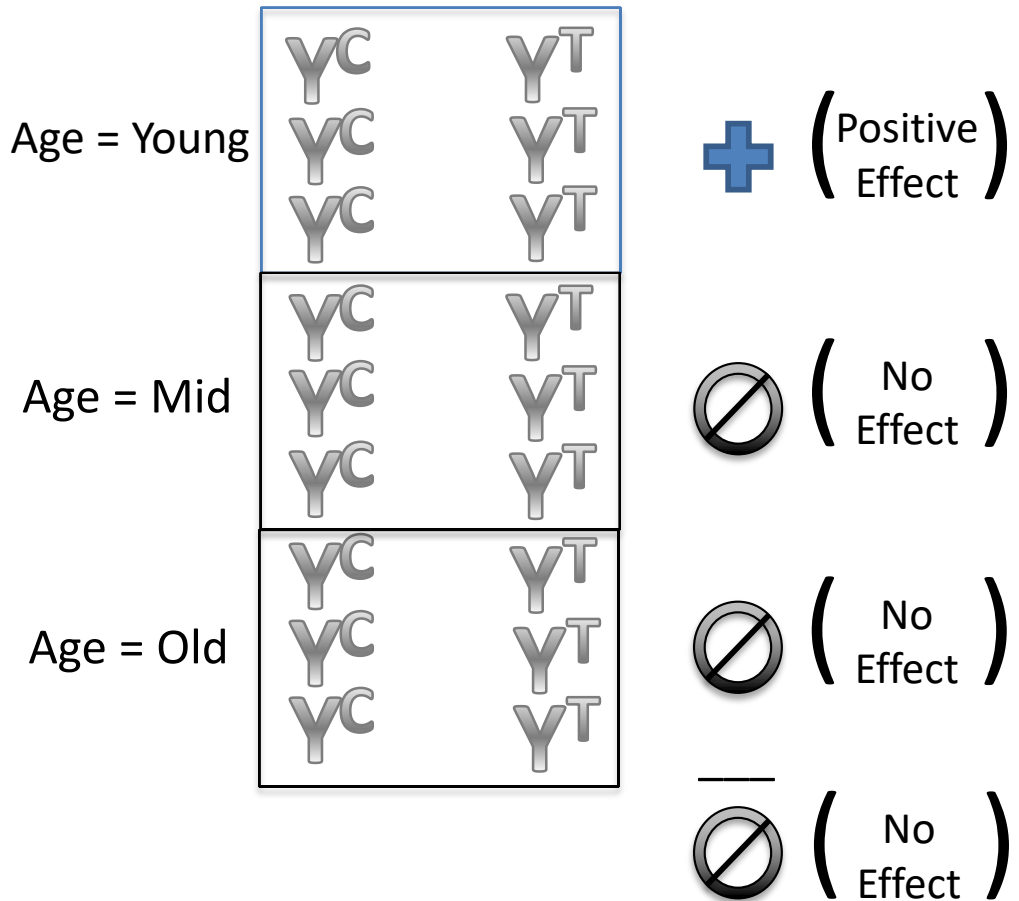
Control Group Treatment Group

Age = Young			 (Positive Effect)
Age = Mid			 (Negative Effect)
Age = Old			 (No Effect)
			 (No Effect)

- Positive and negative effects can cancel

Treatment Effect Heterogeneity





Control Group Treatment Group



- Positive and negative effects can cancel
- True effect can be masked





Treatment Effect Heterogeneity

Control Group Treatment Group

	Control Group	Treatment Group	
Age = Young	YC YC YC	YT YT YT	 (Very Positive Effect)
Age = Mid	YC YC YC	YT YT YT	 (No Effect)
Age = Old	YC YC YC	YT YT YT	 (No Effect)
			 (Slight Effect)

- Positive and negative effects can cancel
- True effect can be masked
- Effects could really be driven by a subpopulation

Treatment Effect Heterogeneity

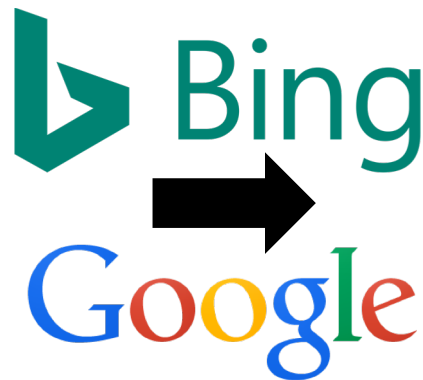
	Control Group	Treatment Group	
Age = Young	YC YC YC	YT YT YT	 (Very Positive Effect)
Age = Mid	YC YC YC	YT YT YT	 (No Effect)
Age = Old	YC YC YC	YT YT YT	 (No Effect)
			 (Slight Effect)

- Positive and negative effects can cancel
- True effect can be masked
 - Ex: FDA Approved BiDil Drug
- Effects could really be driven by a subpopulation
 - Ex: Perry Preschool

Treatment Effect Heterogeneity

Control Group Treatment Group

YC	YT
YC	YT
YC	YT
YC	YT
YC	YT
YC	YT
YC	YT
YC	YT



YC	YC	YC	YC	YC	YC	YC	YC
YT	YT	YT	YT	YT	YT	YT	YT
YC	YC	YC	YC	YC	YC	YC	YC
YT	YT	YT	YT	YT	YT	YT	YT
YC	YC	YC	YC	YC	YC	YC	YC
YT	YT	YT	YT	YT	YT	YT	YT
YC	YC	YC	YC	YC	YC	YC	YC
YT	YT	YT	YT	YT	YT	YT	YT

⋮

YC	YC	YC	YC	YC	YC	YC	YC
YT	YT	YT	YT	YT	YT	YT	YT

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Machine Learning's Contributions

- Regression Methods
 - OLS and Regularized Regression (e.g., LASSO)*
 - Imai and Ratkovic (2013)
 - Taddy et al. (2016)
- Single Tree Methods
 - Su et al (2009)
 - Imai and Strauss (2011)
 - Athey and Imbens (2017)*
- Ensemble Methods
 - Grimmer et al. (2017)
 - Wager and Athey (2017)*

* provide frequentist asym confidence intervals, for inference of effects significance 10

Limitations

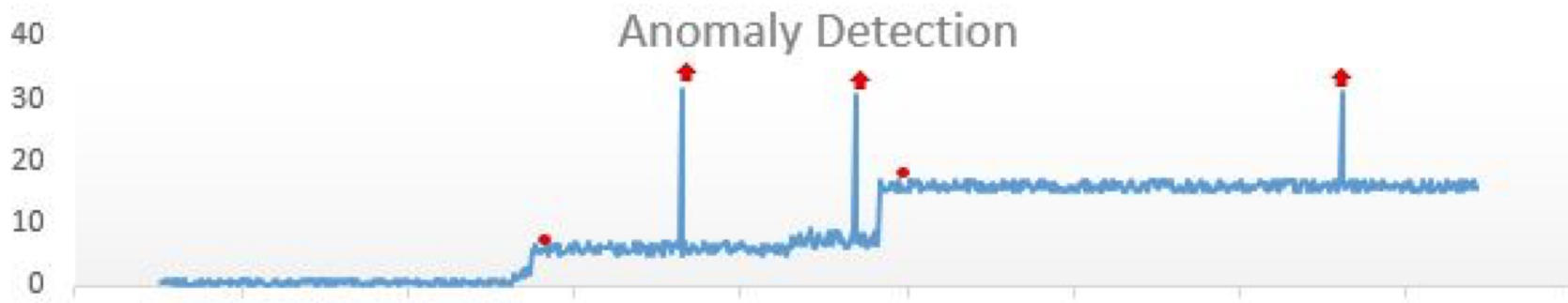
- Regression Methods
 - Pre-specification of the model
- Single Tree Methods
 - Greedy and unstable
- Ensemble Methods
 - Fairly uninterpretable/no natural subpopulations
- General Limitations
 - The mean and only the mean
 - Other moments can be effected
 - Simpsons Paradox
 - Risk minimization not effect maximization
 - Small number of subpopulations considered
 - No guarantee on their “interestingness”
 - No “discovery”, only model inspection

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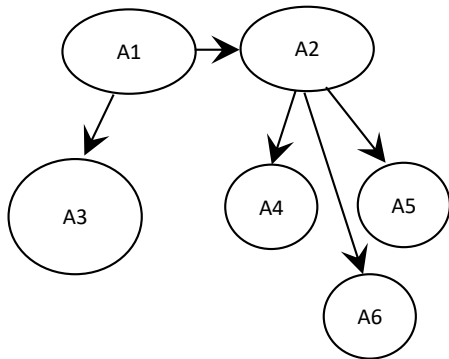
Anomaly Detection Paradigm

- Identifying when a “system” deviates away from its expected behavior.



Anomalous Pattern Detection Procedure

Normal Activity (M_0)



PORT	USPORT	COUNTRY	SLINE	VESSEL	SHIPPERNAME	FNAME	COMMOITY	SIZE	MTONS	VALUE
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	AMERICAN_TRU_NET_EXPRESS/NET	EMPTY_BOX		0	50	2870
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	ORDER	ORDER_OUSED_TRE		2	13.43	947
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	ORDER	ORDER_OUSED_TRE		2	13.43	947
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	AMERICAN_TRU_NET_EXPRESS/NET	CRUDE_OOKE_PURITY		1	17.00	28151
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	NEW_WAVE_TRANSPORT	JT	PAWEL'S_F_MODEL_98	3	30.57	6516
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	NEW_WAVE_TRANSPORT	JT	PAWEL'S_F_MODEL_98	3	30.57	6516
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	NEW_WAVE_TRANSPORT	JT	PAWEL'S_F_MODEL_98	3	30.57	6516
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	ORDER	ORDER_OUSED_TRES		2	13.43	947
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	CHINA_OCEAN_SPPG	CHINA_OCCUPITY_CONTAINERS		0	0	0
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	CHINA_OCEAN_SPPG	CHINA_OCCUPITY_CONTAINERS		0	0	0

Test Data

Detect Anomalous
Pattern Given
 M_0

Novel Pattern

YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	ORDER	ORDER_OUSED_TRE		2	13.43	947
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	ORDER	ORDER_OUSED_TRE		2	13.43	947
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	AMERICAN_TRU_NET_EXPRESS/NET	CRUDE_OOKE_PURITY		1	17.00	28151
YOKOHAMA	SEATTLE	JAPAN	OSSO	LING_YUN_HE	NEW_WAVE_TRANSPORT	JT	PAWEL'S_F_MODEL_98	3	30.57	6516

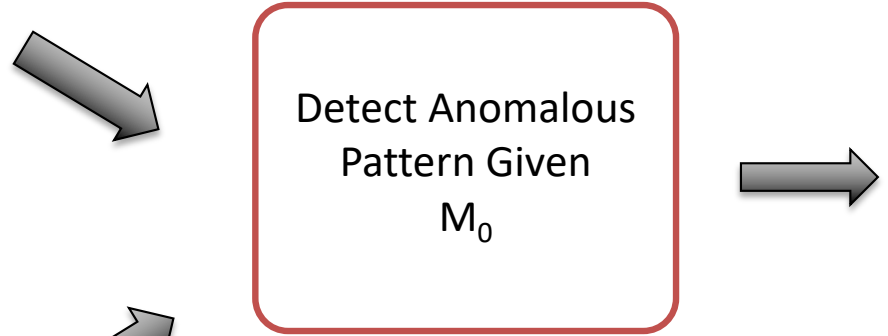
HTE Pattern Detection

Control Group

Column	Order	Carrier	Class	Subclass	Code	Remarks	Remarks	Remarks
1000	3	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	4	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	5	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	6	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	7	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	8	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	9	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	10	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	11	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	12	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	13	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	14	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	15	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	16	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	17	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	18	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	19	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	20	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER

Treatment Group

Column	Order	Carrier	Class	Subclass	Code	Remarks	Remarks	Remarks
1000	3	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	4	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	5	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	6	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	7	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	8	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	9	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	10	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	11	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	12	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	13	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	14	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	15	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	16	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	17	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	18	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	19	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER
1000	20	1000	LINE	YON	ORDER	ORDER	ORDER	ORDER



Novel Pattern

YOKOHAMA	SEATTLE	JAPAN	CSOO	LINE	YON	FE	ORDER	ORDER	ORDER	THE	2	13.43	9407			
YOKOHAMA	SEATTLE	JAPAN	CSOO	LINE	YON	FE	ORDER	ORDER	ORDER	THE	2	13.43	9407			
YOKOHAMA	SEATTLE	JAPAN	CSOO	LINE	YON	FE	AMERICAN	TRAVEL	EXPRESS	NET	CRUISE	CONG	PARTY	1	17.04	28151
YOKOHAMA	SEATTLE	JAPAN	CSOO	LINE	YON	FE	NEW	LINE	TRANSPORT	JT	PAUSE	FE	MODEL	3	26.57	66193

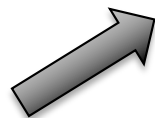
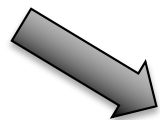
HTE Pattern Detection

Control Group

Column	Header	Row	Applid	SIZE	FEEL	SUPPLEMENT	FAVOR	COMMENTS
100M	B	1000	LAN UN-6	AMERICA TO US EXPRESS NET	EMPTY PACK			
100F	W	4000	LAN UN-6	OVER	OVER DUES THE			
100M	B	4000	LAN UN-6	OVER	OVER DUES THE			
100F	A	1000	LAN UN-6	AMERICA TO US EXPRESS NET	OVER DUES PARTY			
100M	A	4000	LAN UN-6	FOR THE THROAT AT	PLEASE NOTE B			
100M	B	1000	LAN UN-6	FOR THE THROAT AT	PLEASE NOTE B			
100F	W	1000	LAN UN-6	FOR THE THROAT AT	PLEASE NOTE B			
100M	W	1000	LAN UN-6	OVER	OVER DUES THE			
100F	W	2000	LAN UN-6	FOR US UN-6	FOR US UN-6			
100M	B	1000	LAN UN-6	FOR US UN-6	FOR US UN-6			

Treatment Group

Column	Header	Row	Applid	SIZE	FEEL	SUPPLEMENT	FAVOR	COMMENTS
100M	B	1000	LAN UN-6	AMERICA TO US EXPRESS NET	EMPTY PACK			
100F	W	4000	LAN UN-6	OVER	OVER DUES THE			
100M	B	4000	LAN UN-6	OVER	OVER DUES THE			
100F	A	1000	LAN UN-6	AMERICA TO US EXPRESS NET	OVER DUES PARTY			
100M	A	4000	LAN UN-6	FOR THE THROAT AT	PLEASE NOTE B			
100M	B	1000	LAN UN-6	FOR THE THROAT AT	PLEASE NOTE B			
100F	W	1000	LAN UN-6	FOR THE THROAT AT	PLEASE NOTE B			
100M	W	1000	LAN UN-6	OVER	OVER DUES THE			
100F	W	2000	LAN UN-6	FOR US UN-6	FOR US UN-6			
100M	B	1000	LAN UN-6	FOR US UN-6	FOR US UN-6			



Detect Anomalous Subpopulation Given M_0



Subpopulation

100M	B	1000	LAN UN-6	AMERICA TO US EXPRESS NET	EMPTY PACK			
200M	B	2000	LAN UN-6	AMERICA TO US EXPRESS NET	EMPTY PACK			
200M	B	2000	LAN UN-6	AMERICA TO US EXPRESS NET	EMPTY PACK			
100M	B	1000	LAN UN-6	AMERICA TO US EXPRESS NET	EMPTY PACK			
100M	B	1000	LAN UN-6	AMERICA TO US EXPRESS NET	EMPTY PACK			

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The Goal

	Male	Female
Black		
White		
Hispanic		
Asian		
Native American		
Other		

Detect a subpopulation (subsets of attribute values), which correspond to anomalous outcomes for subjects in the treatment group

The Goal

	Male	Female
Black		
White		
Hispanic		
Asian		
Native American		
Other		

Detect a subpopulation (subsets of attribute values), which correspond to anomalous outcomes for subjects in the treatment group

The Optimization

$$s_1 \subseteq \{a_1 \dots a_t\}, \dots, s_M \subseteq \{a_1 \dots a_p\}$$

The Goal

	Male	Female
Black		
White		
Hispanic		
Asian		
Native American		
Other		

Detect a subpopulation (subsets of attribute values), which correspond to anomalous outcomes for subjects in the treatment group

The Optimization

$$s_1 \subseteq \{a_1 \dots a_t\}, \dots, s_M \subseteq \{a_1 \dots a_p\}$$
$$S = s_1 \times \dots \times s_M$$

The Goal

	Male	Female
Black		
White		
Hispanic		
Asian		
Native American		
Other		

Detect a subpopulation (subsets of attribute values), which correspond to anomalous outcomes for subjects in the treatment group

The Optimization

$$s_1 \subseteq \{a_1 \dots a_t\}, \dots, s_M \subseteq \{a_1 \dots a_p\}$$

$$S = s_1 \times \dots \times s_M$$

$$S^* = \operatorname{argmax}_S F(S)$$

Treatment Effects Subset Scan (TESS)

	Male	Female
Black	γ^{BM}	γ^{BF}
White	γ^{WM}	γ^{WF}
Hispanic	γ^{HM}	γ^{HF}
Asian	γ^{AM}	γ^{AF}
Native American	γ^{NM}	γ^{NF}
Other	γ^{OM}	γ^{OF}

I. Compute the statistical anomalousness of each treatment group subject

II. Detect subpopulation that is collectively the most anomalous

Treatment Effects Subset Scan (TESS)

	Male	Female
Black	p^{BM}	p^{BF}
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Hispanic	p^{HM}	p^{HF}
Asian	p^{AM}	p^{AF}
Native American	p^{NM}	p^{NF}
Other	p^{OM}	p^{OF}

I. Compute the statistical anomalousness of each treatment group subject
-- **This measurement will be a p-value**

II. Detect subpopulation that is collectively the most anomalous
-- **Many subjects with significant p-values**

Treatment Effects Subset Scan (TESS)

	Male	Female
Black	γ^{BM}	γ^{BF}
White	γ^{WM}	γ^{WF}
Hispanic	γ^{HM}	γ^{HF}
Asian	γ^{AM}	γ^{AF}
Native American	γ^{NM}	γ^{NF}
Other	γ^{OM}	γ^{OF}
	Control Group	

- I. Compute the statistical anomalousness of each treatment group subject
 1. Estimate Conditional Distribution Under H_0

Treatment Effects Subset Scan (TESS)

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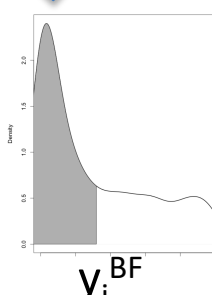
Treatment Group

- I. Compute the statistical anomalousness of each treatment group subject
 1. Estimate Conditional Distribution Under H_0
 2. Compute empirical p-values

Treatment Effects Subset Scan (TESS)

	Male	Female
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Treatment Group



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Treatment Group

- I. Compute the statistical anomalousness of each treatment group subject
 1. Estimate Conditional Distribution Under H_0
 2. Compute empirical p-values
 - i. Maps each bin's distribution to the same interval

Treatment Effects Subset Scan (TESS)

	Male	Female
Black	p^{BM}	p^{BF}
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Asian	p^{AM}	p^{AF}
Native American	p^{NM}	p^{NF}
Other	p^{OM}	p^{OF}

Treatment Group

- I. Compute the statistical anomalousness of each treatment group subject
 1. Estimate Conditional Distribution Under H_0
 2. Compute empirical p-values
 - i. Maps each bin's distribution to the same interval
 - ii. $P_{ij} \sim \text{Uniform}[0,1]$ under H_0

Treatment Effects Subset Scan (TESS)

	Male	Female
Black	p^{BM}	p^{BF}
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Hispanic	p^{HM}	p^{HF}
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Treatment Group

- I. Compute the statistical anomalousness of each treatment group subject
 1. Estimate Conditional Distribution Under H_0
 2. Compute empirical p-values
 - i. Maps each bin's distribution to the same interval
 - ii. $P_{ij} \sim \text{Uniform}[0,1]$ under H_0
 - iii. For any N p-values, we expect $N*\alpha$ to be significant at level α

Treatment Effects Subset Scan (TESS)

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Higher Criticism:

$$F(S) = \max_{\alpha} \frac{N_{\alpha} - N\alpha}{\sqrt{N\alpha(1-\alpha)}}$$

Treatment Effects Subset Scan (TESS)

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Other	p^{OM}	p^{OF}

Treatment Group

- I. Compute the statistical anomalousness of each treatment group subject
- II. Discover subsets of attribute values that define the most anomalous outcomes
 1. Maximize $F(S)$ over all subsets of $S_1 \times \dots \times S_M$
 - Naïve search is infeasible $O(2^{\sum |A_i|})$

Treatment Effects Subset Scan (TESS)

Nonparametric Scan Statistic (NPSS)

$$\begin{aligned}\text{Have: } S &\subseteq \{A_1 \times \dots \times A_M\} \\ &= \{s_1 \times \dots \times s_M\}\end{aligned}$$

- I. Compute the statistical anomalousness of each treatment group subject
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 - Naïve search is infeasible $O(2^{\sum |A_i|})$

Treatment Effects Subset Scan (TESS)

Nonparametric Scan Statistic (NPSS)

Have: $S \subseteq \{A_1 \times \dots \times A_M\}$
 $= \{s_1 \times \dots \times s_M\}$

Select: $F(S) = \max_{\alpha} \phi(\alpha, N_{\alpha}(S), N(S))$

Want: $\max_S F(S)$

Assume: $\phi \uparrow$ w.r.t N_{α}
 $\phi \downarrow$ w.r.t N and α
 ϕ is convex

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Nonparametric Scan Statistic (NPSS)

Have: $S \subseteq \{A_1 \times \dots \times A_M\}$
 $= \{s_1 \times \dots \times s_M\}$

Select: $F(S) = \max_{\alpha} \phi(\alpha, N_{\alpha}(S), N(S))$

Want: $\max_S F(S)$

Assume: $\phi \uparrow$ w.r.t N_{α}
 $\phi \downarrow$ w.r.t N and α
 ϕ is convex

There Exist: $G(a_j)$

Such That: $\max_{s_j \subseteq \{a_1, \dots, a_t\}} F(s_j | A_{-j}) = \max_{i=1 \dots t} F(\{a_{(1)} \dots a_{(t)}\} | A_{-j})$

Only Consider: $\{a_{(1)}\}$
 $\{a_{(1)}, a_{(2)}\}$
 \vdots
 $\{a_{(1)}, \dots, a_{(M)}\}$

- I. Compute the statistical anomalousness of each treatment group subject
- II. Discover subsets of attribute values that define the most anomalous outcomes
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 - Naïve search is infeasible $O(2^{\sum |A_i|})$

Treatment Effects Subset Scan (TESS)

Nonparametric Scan Statistic (NPSS)

Have: $S \subseteq \{A_1 \times \dots \times A_M\}$
 $= \{s_1 \times \dots \times s_M\}$

Select: $F(S) = \max_{\alpha} \phi(\alpha, N_{\alpha}(S), N(S))$

Want: $\max_S F(S)$

Assume: $\phi \uparrow$ w.r.t N_{α}
 $\phi \downarrow$ w.r.t N and α
 ϕ is convex

There Exist: $G(a_j)$

Such That: $\max_{s_j \subseteq \{a_1, \dots, a_t\}} F(s_j | A_{-j}) = \max_{i=1 \dots t} F(\{a_{(1)} \dots a_{(t)}\} | A_{-j})$

Only Consider: {Black}

{Black, Hispanic}

⋮

{Black, Hispanic, Asian, ..., White }

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- II. Discover subsets of attribute values that define the most anomalous outcomes
 1. Maximize $F(S)$ over all subsets of $s_1 \times \dots \times s_M$
 - Naïve search is infeasible $O(2^{\sum |A_i|})$

Treatment Effects Subset Scan (TESS)

Nonparametric Scan Statistic (NPSS)

Have: $S \subseteq \{A_1 \times \dots \times A_M\}$
 $= \{s_1 \times \dots \times s_M\}$

Select: $F(S) = \max_{\alpha} \phi(\alpha, N_{\alpha}(S), N(S))$

Want: $\max_S F(S)$

Assume: $\phi \uparrow$ w.r.t N_{α}
 $\phi \downarrow$ w.r.t N and α
 ϕ is convex

There Exist: $G(a_j) = \frac{1}{n(a_j)} \sum_{A_{-j}} I(p_{ij} \leq \alpha)$

Such That: $\max_{s_j \subseteq \{a_1, \dots, a_t\}} F(s_j | A_{-j}) = \max_{i=1 \dots t} F(\{a_{(1)} \dots a_{(t)}\} | A_{-j})$

Only Consider: {Black}

{Black, Hispanic}

⋮

{Black, Hispanic, Asian, ..., White }

- I. Compute the statistical anomalousness of each treatment group subject
- II. Discover subsets of attribute values that define the most anomalous outcomes
 1. Maximize $F(S)$ over all subsets of $s_1 \times \dots \times s_M$
 - Naïve search is infeasible $O(2^{\sum |A_i|})$

Treatment Effects Subset Scan (TESS)

Nonparametric Scan Statistic (NPSS)

$$\begin{aligned} \text{Have: } S &\subseteq \{A_1 \times \dots \times A_M\} \\ &= \{s_1 \times \dots \times s_M\} \end{aligned}$$

$$\text{Select: } F(S) = \max_{\alpha} \phi(\alpha, N_{\alpha}(S), N(S))$$

$$\text{Want: } \max_S F(S)$$

$$\begin{aligned} \text{Assume: } \phi &\uparrow \text{ w.r.t } N_{\alpha} \\ \phi &\downarrow \text{ w.r.t } N \text{ and } \alpha \\ \phi &\text{ is convex} \end{aligned}$$

$$\text{There Exist: } G(a_j) = \frac{1}{n(a_j)} \sum_{A_{-j}} I(p_{ij} \leq \alpha)$$

$$\text{Such That: } \max_{s_j \subseteq \{a_1, \dots, a_t\}} F(s_j | A_{-j}) = \max_{i=1 \dots t} F(\{a_{(1)} \dots a_{(t)}\} | A_{-j})$$

$$\begin{aligned} \text{Intuitively: } F(\{a_{(1)}, a_{(3)}\}) &\leq F(\{a_{(1)}, a_{(2)}\}) \\ G(a_{(3)}) &\leq G(a_{(2)}) \end{aligned}$$

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Higher Criticism:

$$F(S) = \max_{\alpha} \frac{N_{\alpha} - N\alpha}{\sqrt{N\alpha(1-\alpha)}}$$

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 - NPSS over an attribute in $O(t \log t)$


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TESS Search Procedure

	Male	Female
Black	p^{BM}	p^{BF}
White	p^{WM}	p^{WF}
Hispanic	p^{HM}	p^{HF}
Asian	p^{AM}	p^{AF}
Native American	p^{NM}	p^{NF}
Other	p^{OM}	p^{OF}

Treatment Group



- I. Compute the statistical anomalousness of each treatment group subject
 1. Estimate Conditional Distribution Under H_0
 2. Compute empirical p-values
- II. Discover subsets of attribute values that define the most anomalous outcomes
 1. Maximize $F(S)$ over all subsets of $S_1 \times \dots \times S_M$
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TESS Search Procedure

↓

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(Score = 7.5)

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TESS Search Procedure

↓

	Male	Female	
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White	p^{WM}	p^{WF}	
Hispanic	p^{HM}	p^{HF}	←
Asian	p^{AM}	p^{AF}	
Native American	p^{NM}	p^{NF}	
Other	p^{OM}	p^{OF}	

(Score = 8.1)

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TESS Search Procedure



Male

Female

Black	p^{BM}	p^{BF}	←
White	p^{WM}	p^{WF}	
Hispanic	p^{HM}	p^{HF}	←
Asian	p^{AM}	p^{AF}	
Native American	p^{NM}	p^{NF}	
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(Score = 9.3)

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↓

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Significance of our subpopulation
 Compare subpopulation score to maximum scores of simulated datasets under H_0

Agenda

- Introduction and Motivation
- Machine Learning's Contributions (and Limits)
- How Anomaly Detection Can Help
- **Treatment Effect Subset Scan**
 - Algorithm
 - Statistical Properties
- Results
- Conclusions

TESS's Statistical Properties

- False Positive under H_0 (**Theorem 2**)

$$P_{H_0} \left(\max_S F(S) > h(M) \right) \rightarrow 0$$

- Power under H_1 (**Theorem 3**)

$$P_{H_1} \left(\max_S F(S) > h(M) \right) \rightarrow 1$$

- Exactness of Detected Subpopulation

- Sufficiently Homogenous (**Theorem 4**)

$$S^* \supseteq S^T$$

- Sufficiently Strong (**Theorem 5**)

$$S^* \subseteq S^T$$

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Tennessee Star Analysis (1985)

- Effect of classrooms size on achievement (test scores)
- 4 year panel (kindergarten to 3rd grade)
- 6,500 students, 330 classrooms, 80 schools
 - Total of over 11,000 records
- Treatment Conditions (randomized within school)
 - Regular Size Class (20-25 students)
 - Regular Size + Aide Class (20-25 students)
 - Small Size Class (13-17 students)

Tennessee Star Analysis (1985)

read	math	gender	ethnicity	lunch	grade	school	experience	degree	tethnicity	schoolid
439	463	male	afam	free	kindergarten	inner-city	0	bachelor	cauc	19
448	559	male	cauc	non-free	kindergarten	rural	16	bachelor	cauc	69
431	454	male	cauc	free	kindergarten	rural	8	bachelor	cauc	5
395	423	female	afam	free	kindergarten	inner-city	17	master	cauc	16
451	500	female	cauc	non-free	kindergarten	rural	3	bachelor	afam	56
430	473	male	cauc	non-free	kindergarten	rural	13	master	cauc	38
437	468	male	cauc	non-free	kindergarten	rural	6	master	cauc	69
490	528	male	cauc	non-free	kindergarten	suburban	18	bachelor	cauc	52
439	484	male	cauc	non-free	kindergarten	suburban	13	master	cauc	54
424	459	female	cauc	free	kindergarten	rural	12	bachelor	cauc	12
437	528	female	afam	free	kindergarten	suburban	1	bachelor	afam	21
424	559	male	cauc	free	kindergarten	rural	13	bachelor	cauc	79
431	454	male	cauc	non-free	kindergarten	rural	13	master	cauc	8
451	473	male	cauc	non-free	kindergarten	rural	3	bachelor	cauc	66
421	459	female	afam	free	kindergarten	inner-city	11	bachelor	cauc	31

Tennessee Star Analysis

	(1)	(2)
Treatment	3.4791	-0.2909
	(2.547)	(2.277)
Sample	All 2 nd Grade	All 3 rd Grade
R-squared	0.000	0.000
Observations	4263	4063

Notes: All estimates are from OLS models.

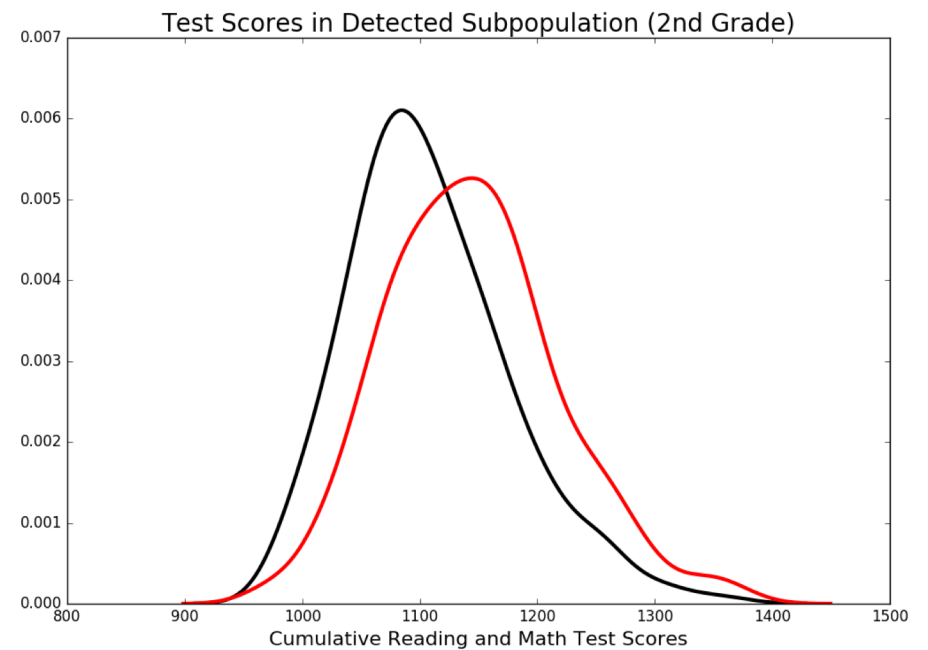
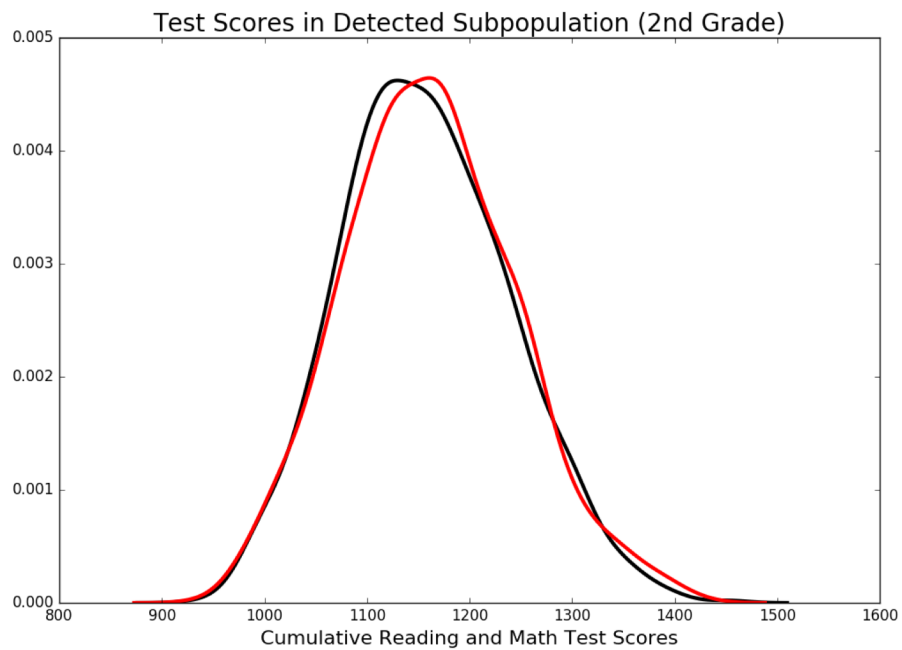
Standard errors are in parentheses.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

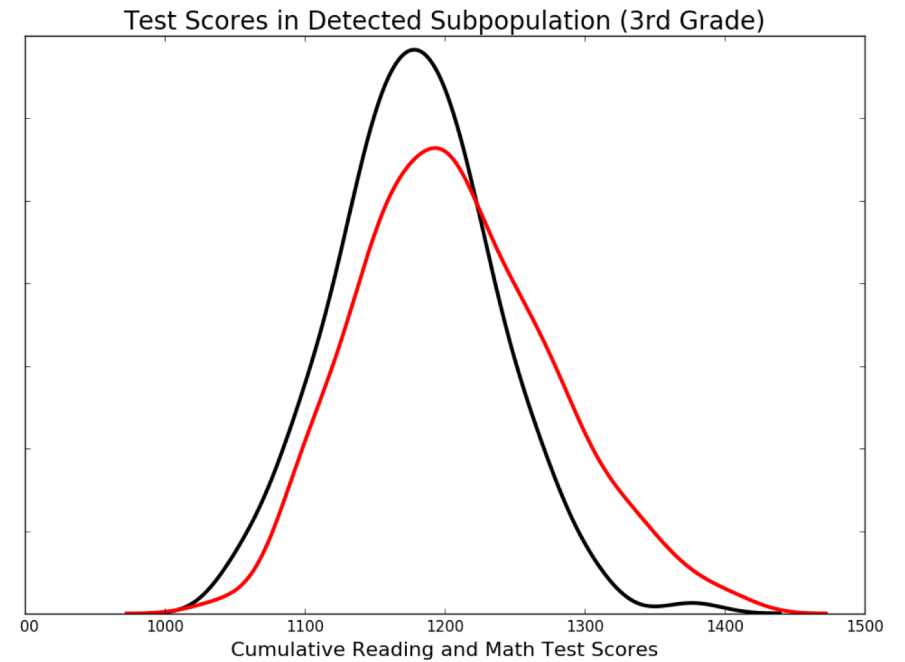
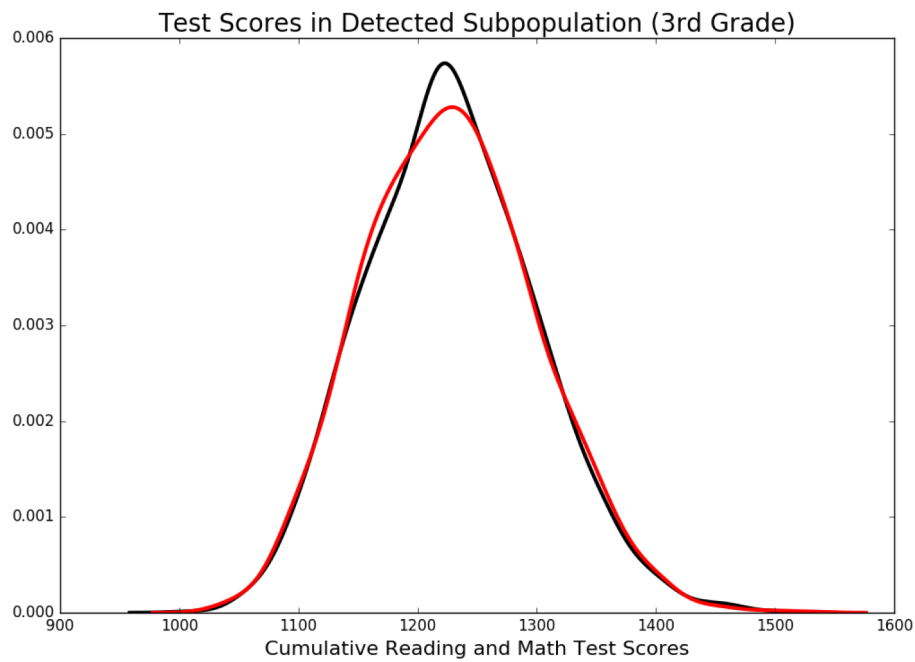
Tennessee Star Analysis (1985)

- Detected Subpopulation
 - grade:
 - 2nd or 3rd
 - school:
 - inner-city or urban
 - experience:
 - [10, infinity)
 - other features do not have differential effects

Tennessee Star Analysis



Tennessee Star Analysis



Tennessee Star Analysis

	(1)	(2)	(3)
Treatment	3.4791	36.066***	1.309
	(2.547)	(6.055)	(2.772)
Sample	All 2 nd Grade	Detected Group (2 nd Grade)	Undetected Group (2 nd Grade)
P-value	0.172	<0.001	0.0637
Observations	4263	620	3643

Notes: All estimates are from OLS models.

Standard errors are in parentheses.

*** p<0.001, ** p<0.05, * p<0.1

Tennessee Star Analysis

	(1)	(2)	(3)
Treatment	-0.291	18.703***	0.1
	(2.277)	(5.18)	(2.478)
Sample	All 3 rd Grade	Detected Group (3 rd Grade)	Undetected Group (3 rd Grade)
P-value	0.898	<0.001	0.968
Observations	4063	706	3357

Notes: All estimates are from OLS models.

Standard errors are in parentheses.

*** p<0.001, ** p<0.05, * p<0.1

Conclusion

- Discovering subpopulations with significant treatment effects can be paramount
- Machine Learning can flexibly estimate effects but it is limited when goal is to identify subpopulations with large effects
- Anomalous Pattern Detection paradigm offers to overcome some of the limitations of Machine Learning
 - Maintain high power to detect by searching over and combining signal across various subpopulations