Scalable Bayesian Event Detection and Visualization

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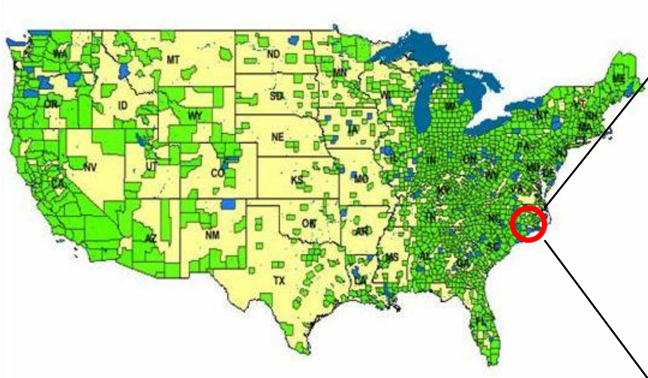
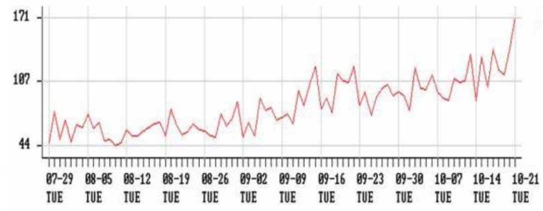
Multivariate event detection



Daily health data from thousands of hospitals and pharmacies nationwide.



Time series of counts $c_{i,m}^t$ for each zip code s_i for each data stream d_m.

d_1 = respiratory ED

d_2 = constitutional ED

d_3 = OTC cough/cold

d_4 = OTC anti-fever

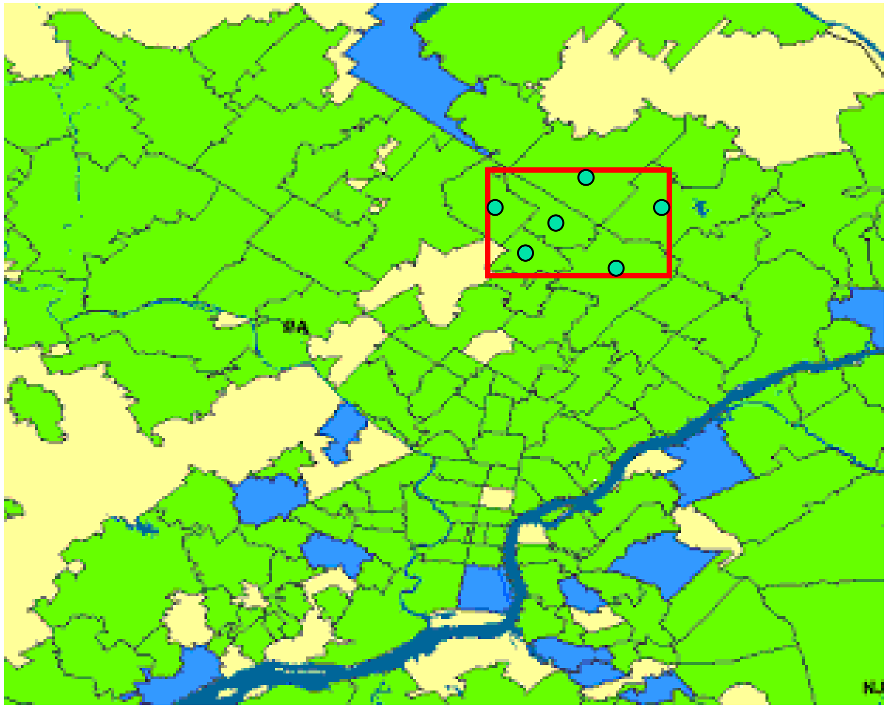
etc.

Given all of this nationwide health data on a daily basis, we want to obtain a complete <u>situational awareness</u> by integrating information from the multiple data streams.

More precisely, we have three main goals: to <u>detect</u> any emerging events (i.e. outbreaks of disease), <u>characterize</u> the type of event, and <u>pinpoint</u> the affected areas.

Expectation-based scan statistics

(Kulldorff, 1997; Neill and Moore, 2005)



To detect and localize events, we can search for <u>space-time regions</u> where the number of cases is higher than expected.

Imagine moving a window around the scan area, allowing the window size, shape, and temporal duration to vary.

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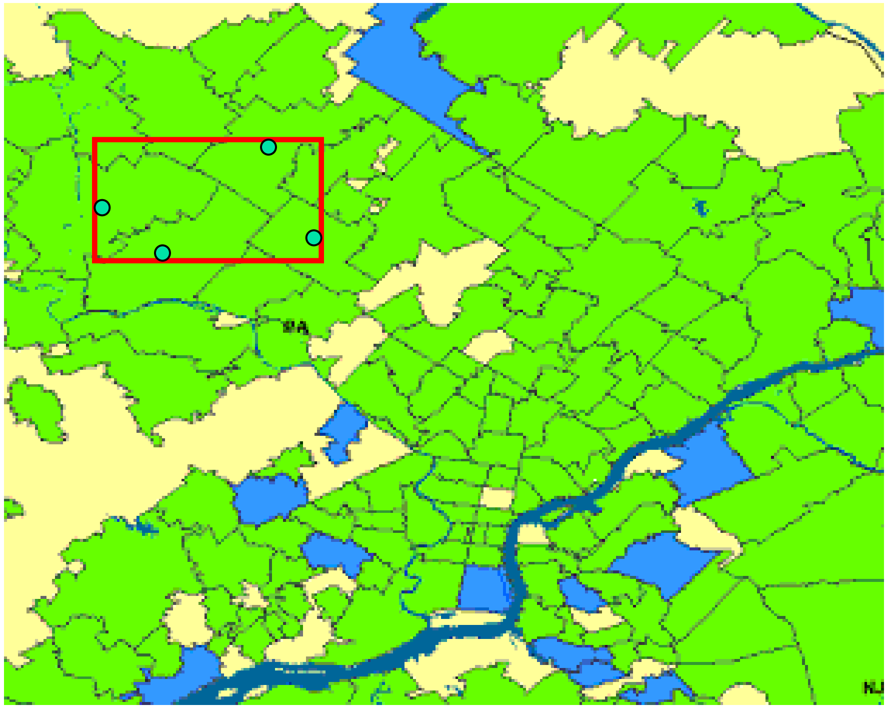

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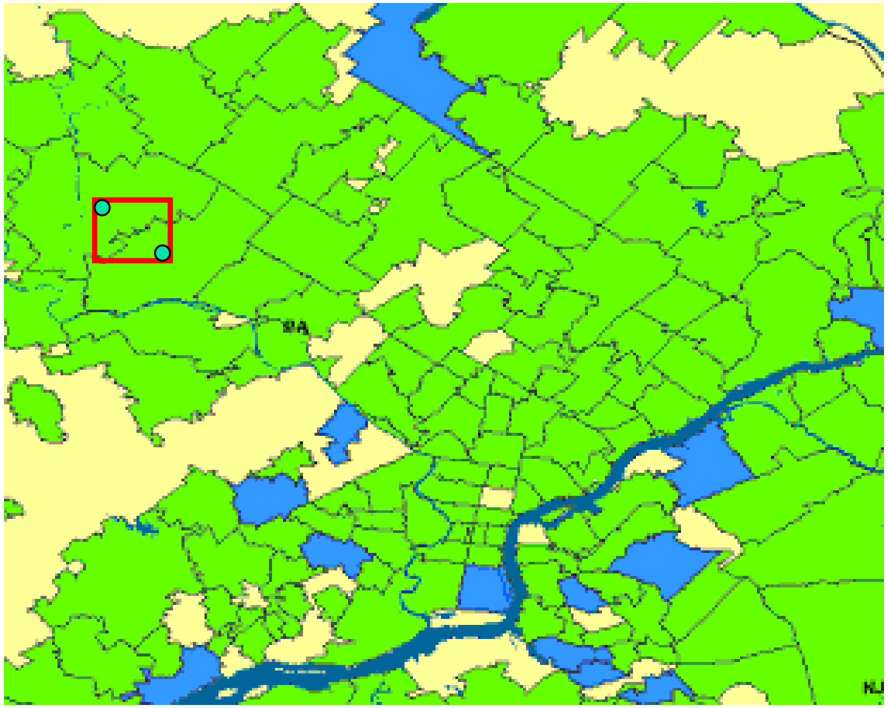

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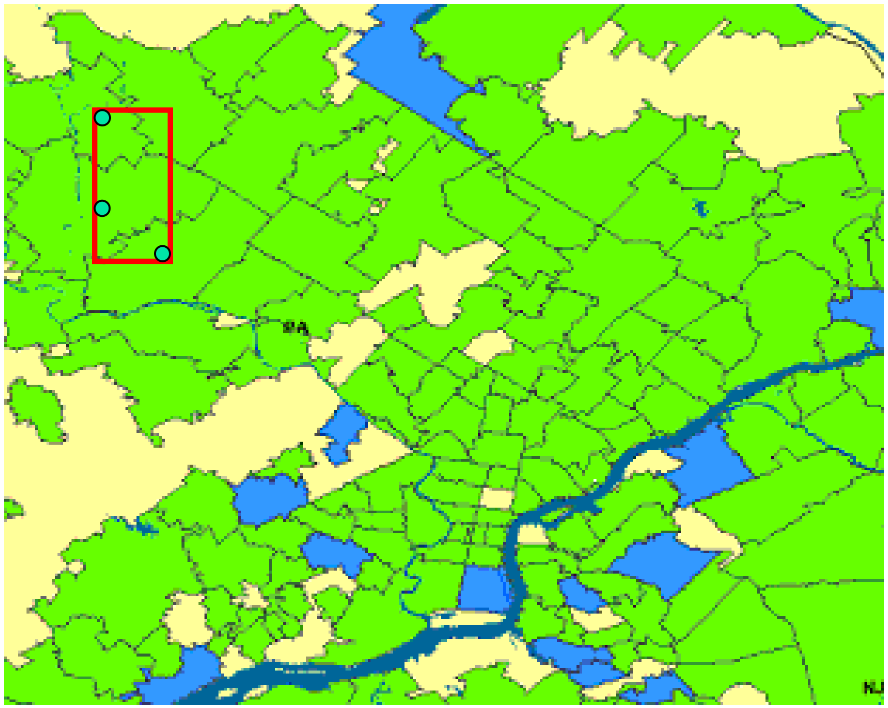

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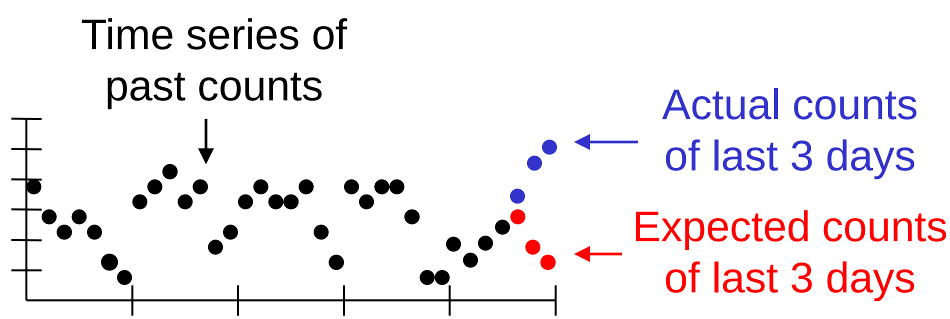
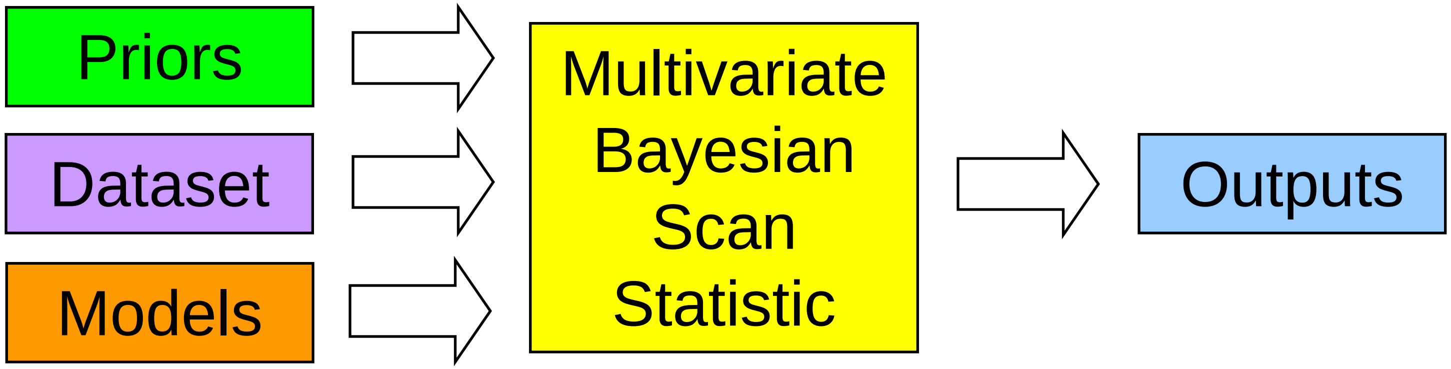
Imagine moving a window around the scan area, allowing the window size, shape, and temporal duration to vary.

For each subset of locations, we examine the aggregated time series, and compare actual to expected counts.



Overview of the MBSS method

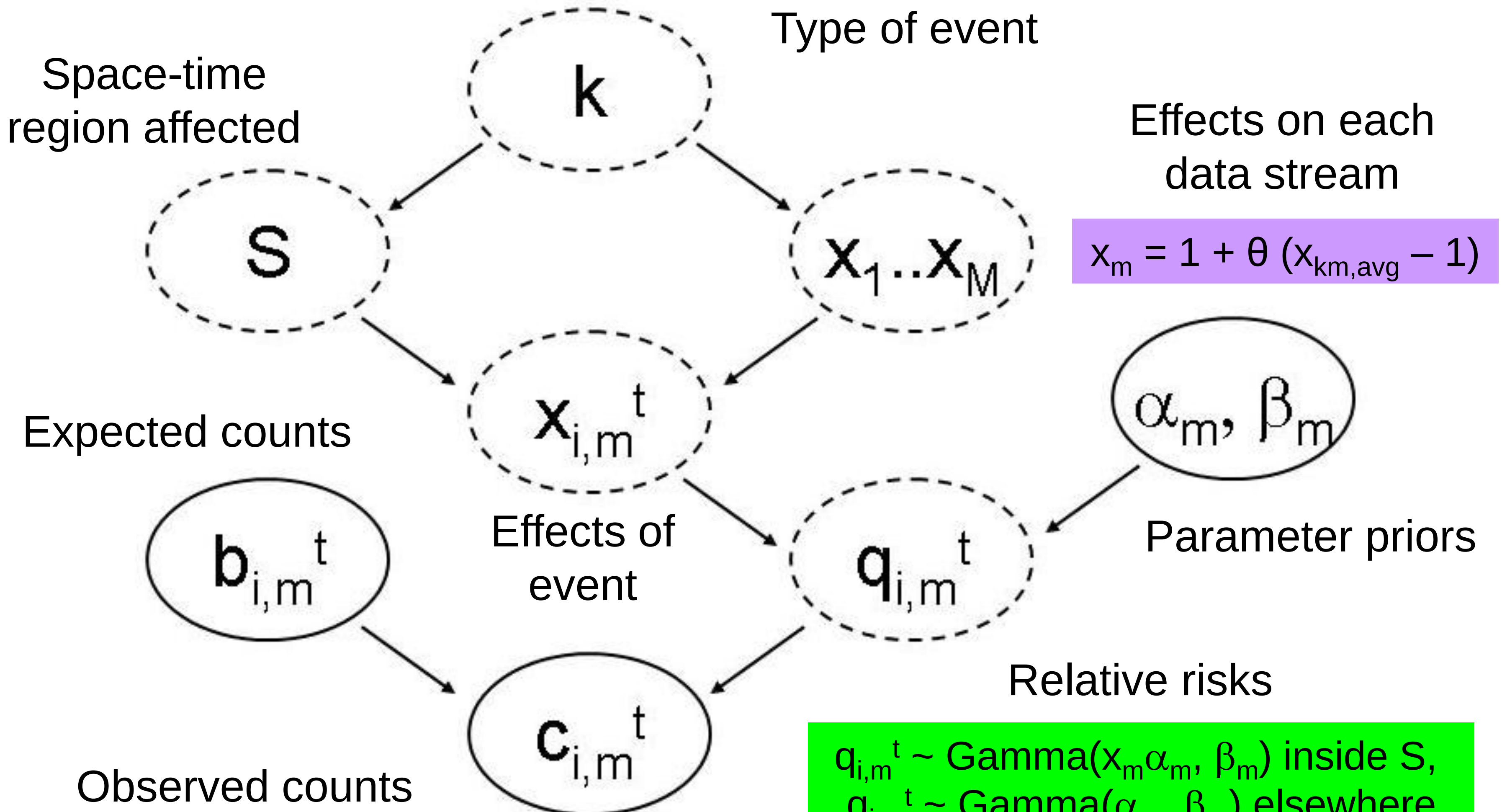


Given a set of event types E_k, a set of space-time regions S, and the multivariate dataset D, MBSS outputs the posterior probability $Pr(H_1(S, E_k) \mid D)$ of each type of event in each region, as well as the probability of no event, $Pr(H_0 \mid D)$.

We must provide the prior probability $Pr(H_1(S, E_k))$ of each event type E_k in each region S, as well as the prior probability of no event, $Pr(H_0)$.

MBSS uses Bayes' Theorem to combine the data likelihood given each hypothesis with the prior probability of that hypothesis: $Pr(H \mid D) = Pr(D \mid H)\, Pr(H) / Pr(D)$.

The Bayesian hierarchical model



$x_m = 1 + \theta\,(x_{km,avg} - 1)$

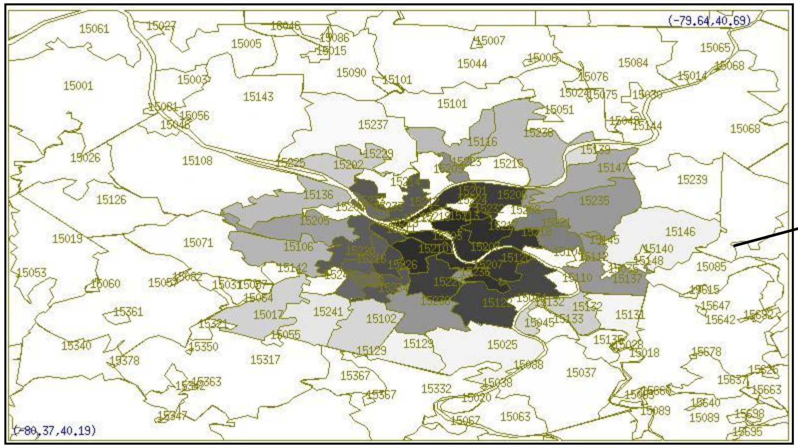
$q_{i,m}^t \sim \text{Gamma}(x_m\alpha_m, \beta_m)$ inside S,
$q_{i,m}^t \sim \text{Gamma}(\alpha_m, \beta_m)$ elsewhere

$c_{i,m}^t \sim \text{Poisson}(q_{i,m}^t b_{i,m}^t)$

Interpretation and visualization

MBSS gives the total posterior probability of each event type E_k, and the distribution of this probability over space-time regions S.

<u>Visualization</u>: $Pr(H_1(s_i, E_k)) = \sum Pr(H_1(S, E_k))$ for all regions S containing location s_i.



<u>Posterior probability map</u>

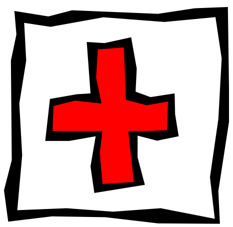
Total posterior probability of a respiratory outbreak in each Allegheny County zip code.

Darker shading = higher probability.

MBSS: advantages and limitations

MBSS can detect faster and more accurately by integrating multiple data streams.

MBSS can model and differentiate between multiple potential causes of an event.





MBSS assumes a uniform prior for circular regions and zero prior for non-circular regions, resulting in low power for **elongated** or **irregular** clusters.

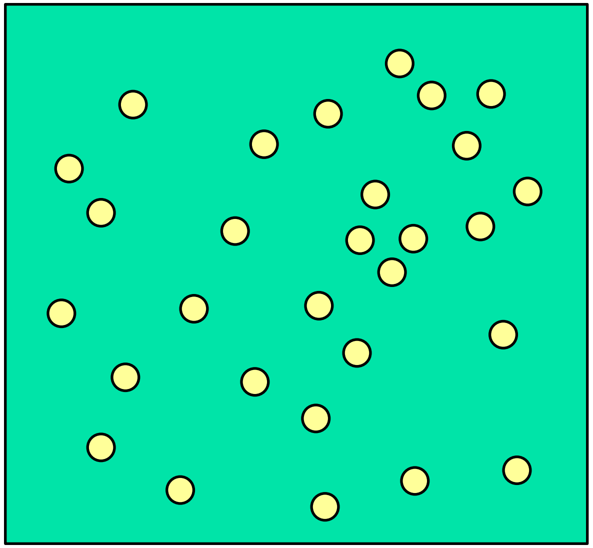
There are too many subsets of the data (2^N) to compute likelihoods for all of them!

How can we extend MBSS to **efficiently** detect irregular clusters?

Hierarchical prior distribution

We define a non-uniform prior $Pr(H_1(S, E_k))$ over all 2^N subsets of the data.

This prior has hierarchical structure:



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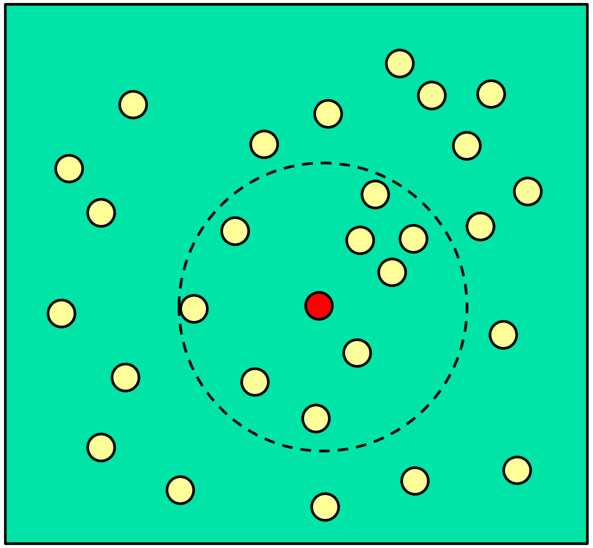
1. Choose the **center location s_c** uniformly at random from $\{s_1 \ldots s_N\}$.
2. Choose the **neighborhood size n** uniformly at random from $\{1 \ldots n_{max}\}$.



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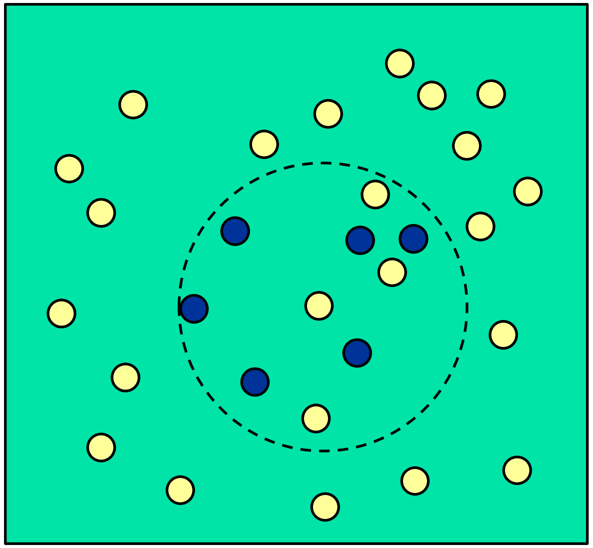
1. Choose the **center location s_c** uniformly at random from $\{s_1 \ldots s_N\}$.
2. Choose the **neighborhood size n** uniformly at random from $\{1 \ldots n_{max}\}$.
3. Choose **region S** uniformly at random from the 2^n subsets of S_{cn} = $\{s_c$ and its n – 1 nearest neighbors$\}$.



This prior distribution has non-zero prior probabilities for any given subset S, but more compact clusters have larger priors.

Fast Subset Sums (FSS)

Naïve computation of posterior probabilities using this prior requires summing over an exponential number of regions, which is infeasible.

However, the total posterior probability of an outbreak, $\Pr(H_1(E_k) \mid D)$, and the posterior probability map, $\Pr(H_1(s_i, E_k) \mid D)$, can be calculated efficiently **without** computing the probability of each region S.

In the original MBSS method, the **likelihood ratio** of spatial region S for a given event type E_k and event severity θ can be found by multiplying the likelihood ratios $LR(s_i \mid E_k, \theta)$ for all locations s_i in S.

In FSS, the **average likelihood ratio** of the 2^n subsets for a given center s_c and neighborhood size n can be found by multiplying the quantities $((1 + LR(s_i \mid E_k, \theta)) / 2)$ for all locations s_i in S.

Since the prior is uniform for a given center and neighborhood, we can compute the posteriors for each s_c and n, and marginalize over them.

Fast Subset Sums (FSS)

<u>Proof sketch</u>: For a given center s_c, neighborhood size n, event type E_k, severity θ, and temporal window W, we can compute the summed posterior probability of all subsets S_{cn}.

$$\sum_{S \subseteq S_{cn}} \Pr(S \mid D) \propto \sum_{S \subseteq S_{cn}} \Pr(S)\, LR(S)$$

$$= (1 / 2^n) \sum_{S \subseteq S_{cn}} LR(S)$$

$$= (1 / 2^n) \sum_{S \subseteq S_{cn}} \prod_{s_i \in S} LR(s_i).$$

<u>Key step</u>: write sum of 2^n products as product of n sums.

$$= (1 / 2^n) \prod_{s_i \in S_{cn}} (1 + LR(s_i))$$

$$= \prod_{s_i \in S_{cn}} ((1 + LR(s_i)) / 2)$$

Fast Subset Sums (FSS)

Proof sketch: To compute the posterior probability map, we must compute the summed posterior probability of all subsets S_{cn} containing each location s_j.

$$\sum_{S \subseteq S_{cn}:\, s_j \in S} \Pr(S \mid D) \propto \sum_{S \subseteq S_{cn}:\, s_j \in S} \Pr(S)\, LR(S)$$

$$= (1 / 2^n) \sum_{S \subseteq S_{cn}:\, s_j \in S} LR(S)$$

$$= (1 / 2^n) \sum_{S \subseteq S_{cn}:\, s_j \in S} \prod_{s_i \in S} LR(s_i).$$

Key step: write sum of 2^{n-1} products as product of n-1 sums.

$$= (1 / 2^n)\, LR(s_j) \prod_{s_i \in S_{cn}-\{s_j\}} (1 + LR(s_i))$$

$$= LR(s_j) / (1 + LR(s_j)) * \sum_{S \subseteq S_{cn}} \Pr(S \mid D)$$

Evaluation

- We injected simulated disease outbreaks into two streams of Emergency Department data (cough, nausea) from 97 Allegheny County zip codes.
- Results were computed for ten different outbreak shapes, including compact, elongated, and irregularly-shaped, with 200 injects of each type.
- We compared FSS to the original MBSS method (searching over circles) in terms of run time, timeliness of detection, proportion of outbreaks detected, and spatial accuracy.

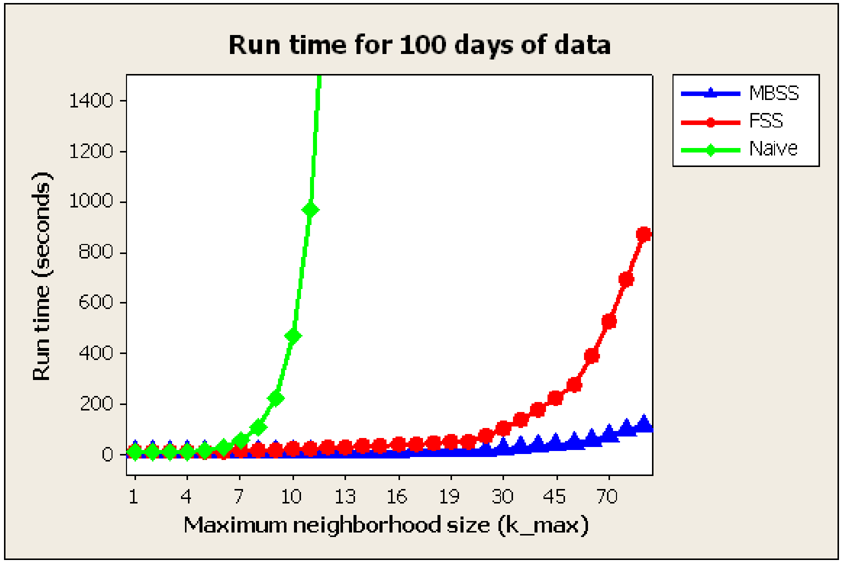
Computation time

We compared the run times of MBSS, FSS, and a naïve subset sums implementation as a function of the maximum neighborhood size n_{max}.

Run time of MBSS increased gradually with increasing n_{max}, up to 1.2 seconds per day of data.

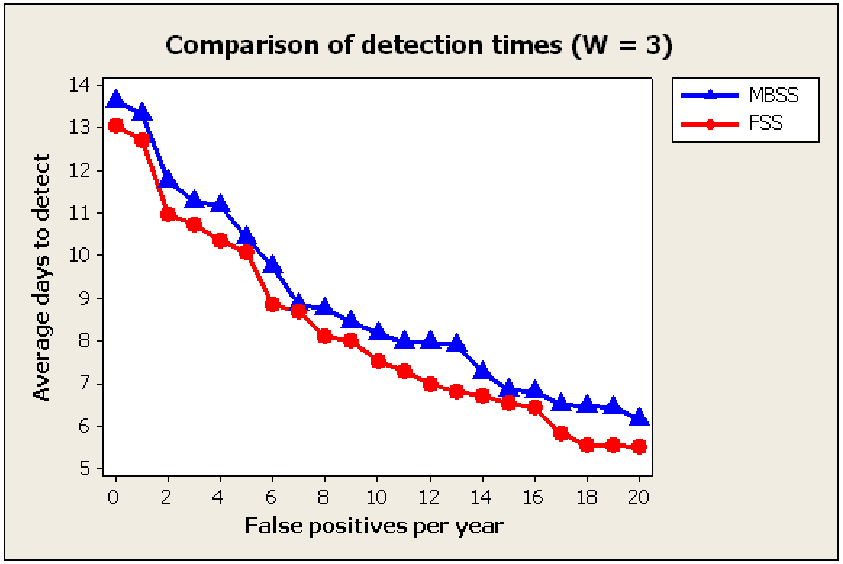
Run time of Naïve Subset Sums increased exponentially, making it infeasible for $n_{max} \geq 25$.

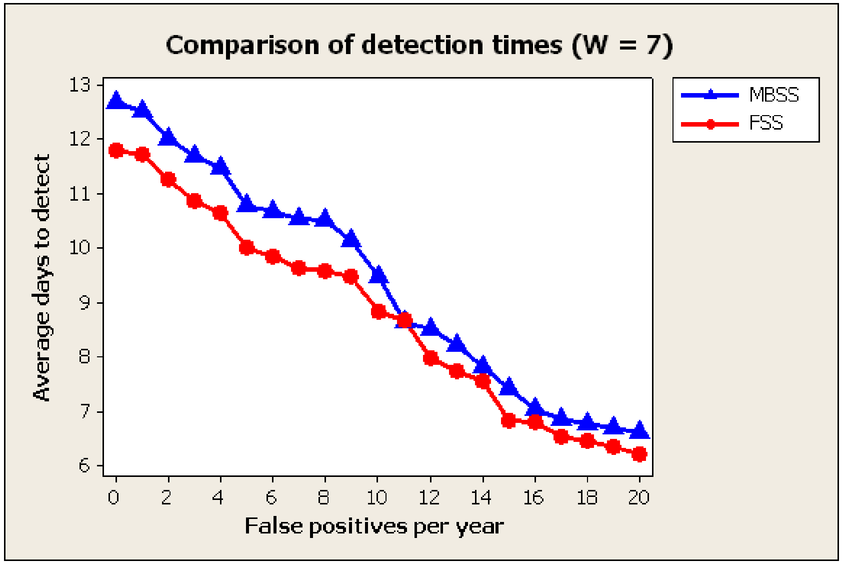
Run time of FSS scaled quadratically with n_{max}, up to 8.8 seconds per day of data.



Thus, while FSS is approximately 7.5x slower than the original MBSS method, it is still extremely fast, computing the posterior probability map for each day of data in under nine seconds.

Timeliness of detection

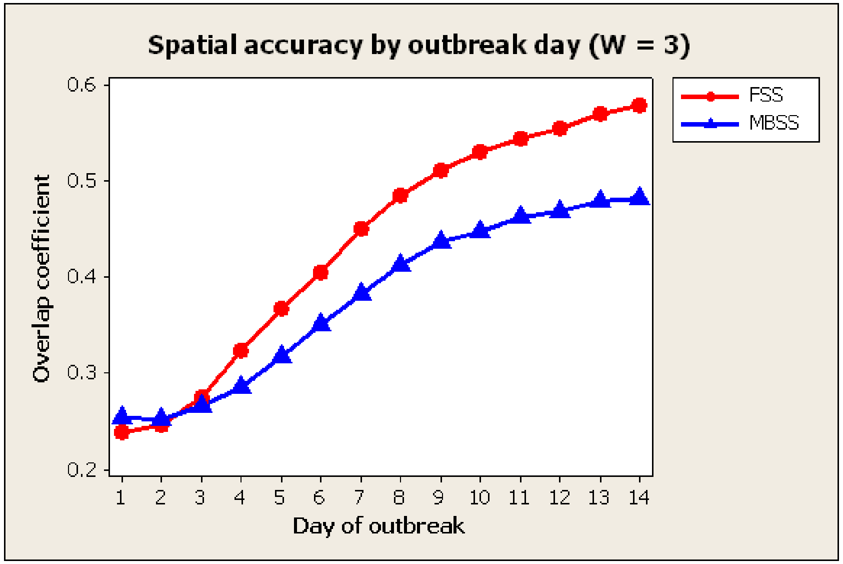


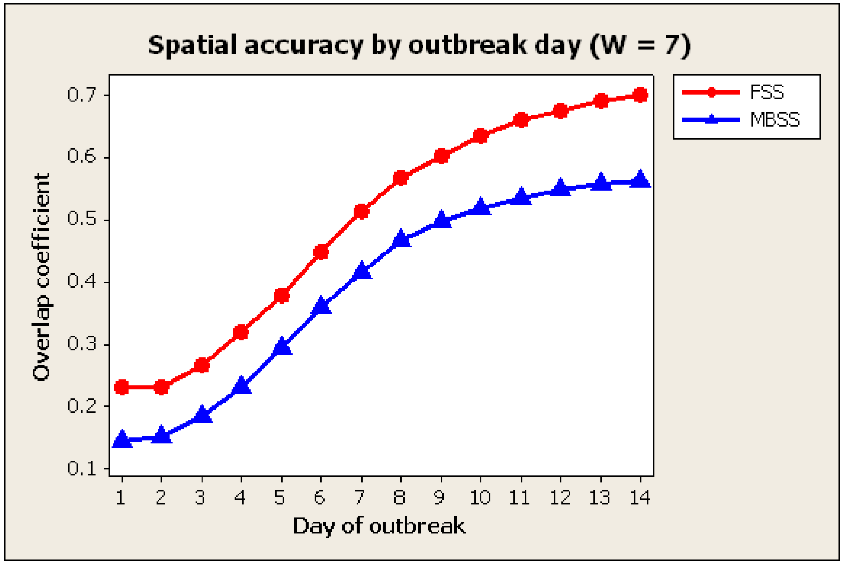


FSS detected an average of **one day earlier** than MBSS for maximum temporal window W = 3, and **0.54 days earlier** for W = 7, with less than half as many missed outbreaks.

Both methods achieve similar detection times for compact outbreak regions. For highly elongated outbreaks, FSS detects 1.3 to 2.2 days earlier, and for irregular regions, FSS detects 0.3 to 1.2 days earlier.

Spatial accuracy





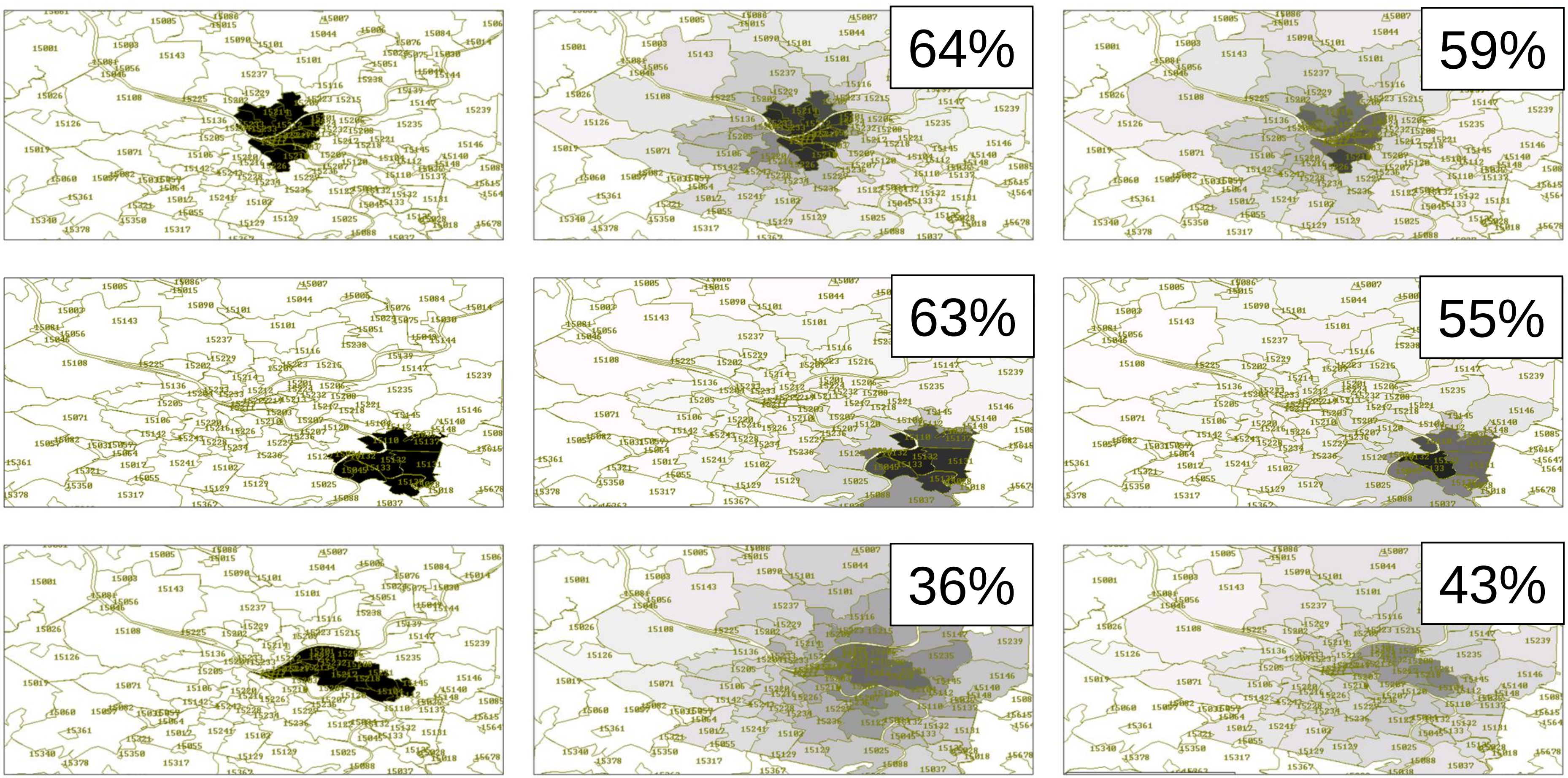
As measured by the average overlap coefficient between true and detected clusters, FSS outperformed MBSS by 10-15%.

For elongated and irregular clusters, FSS had much higher precision and recall. For compact clusters, FSS had higher precision, and MBSS had higher recall.

Posterior probability maps

Spatial accuracy of FSS was similar to MBSS for compact clusters.

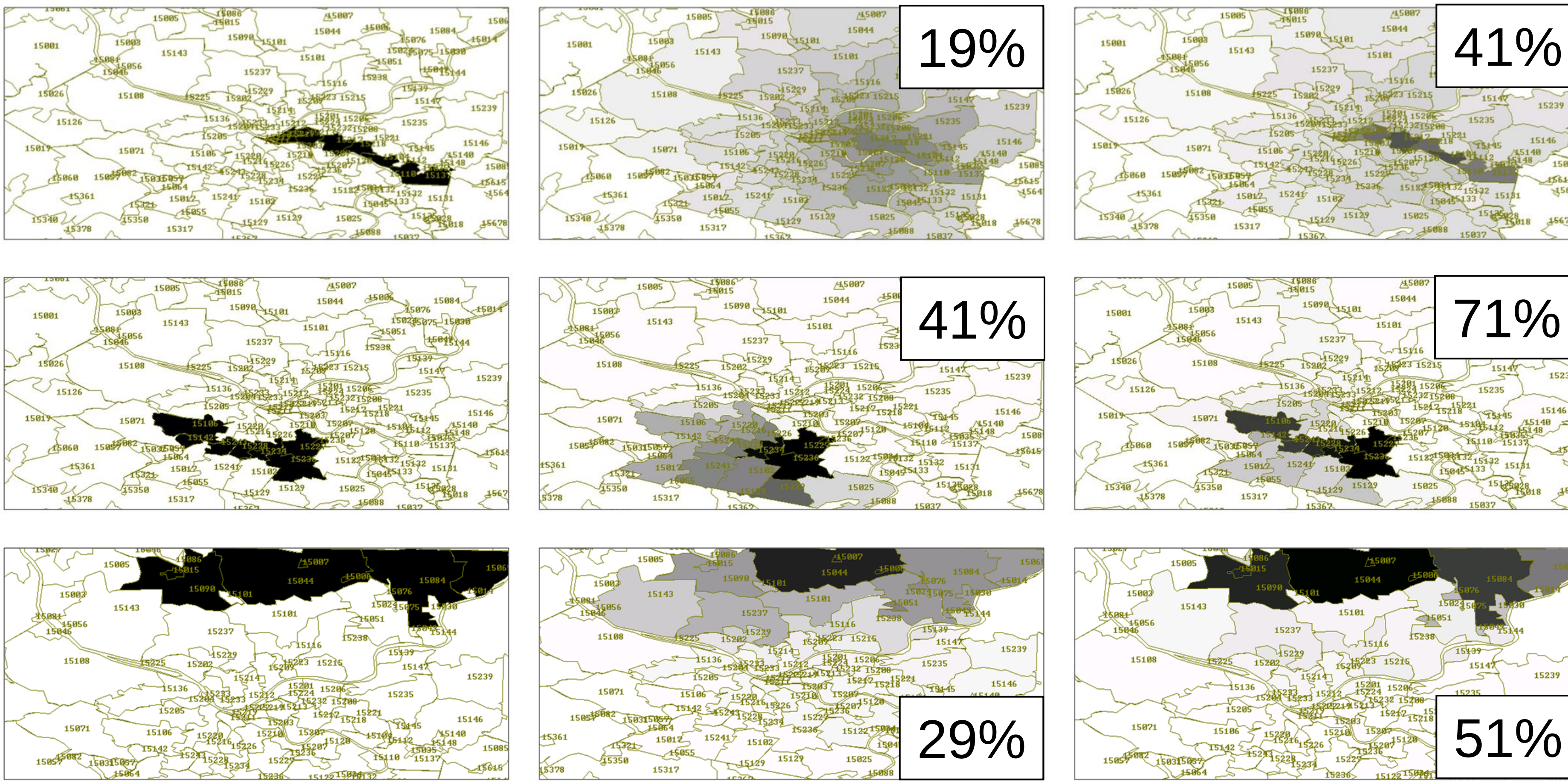
True outbreak region | MBSS | Fast Subset Sums



Posterior probability maps

FSS had much higher spatial accuracy than MBSS for elongated clusters.

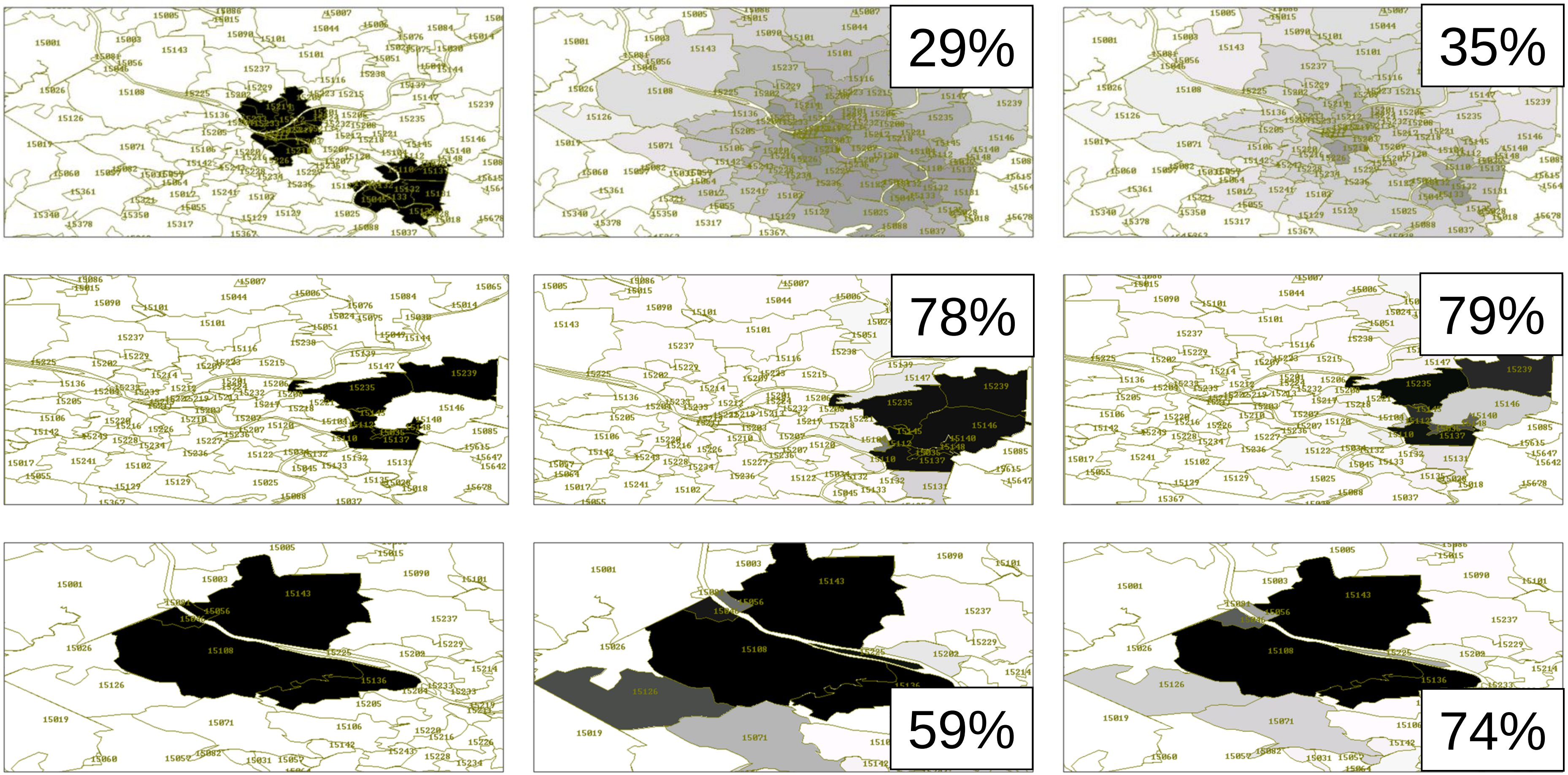
True outbreak region | MBSS | Fast Subset Sums



Posterior probability maps

FSS was better able to capture the shape of irregular clusters.

True outbreak region | MBSS | Fast Subset Sums

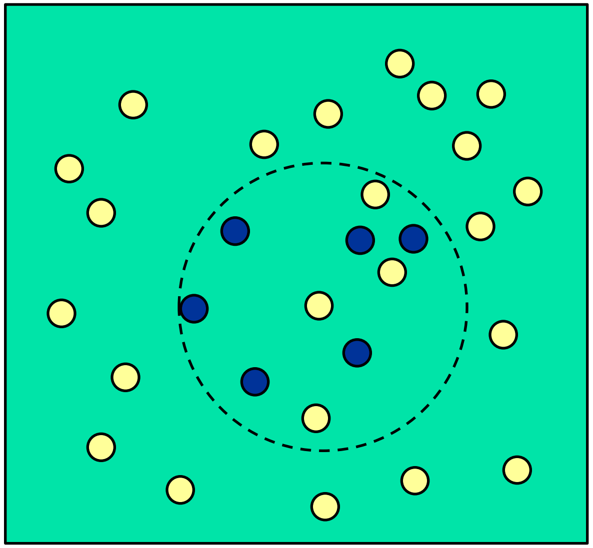


Generalized FSS

We have recently developed a generalization of FSS which allows another parameter (sparsity of the detected region) to be controlled.

We assume a different hierarchical prior:

1. Choose the **center location s_c** uniformly at random from $\{s_1 \ldots s_N\}$.
2. Choose the **neighborhood size n** uniformly at random from $\{1 \ldots n_{max}\}$.
3. For each $s_i \in S_{cn}$, include s_i in S with probability p, for a fixed $0 < p \leq 1$.



p = 0.5 corresponds to the original FSS approach (all subsets equally likely given the neighborhood).

p = 1 corresponds to MBSS (circular regions only).

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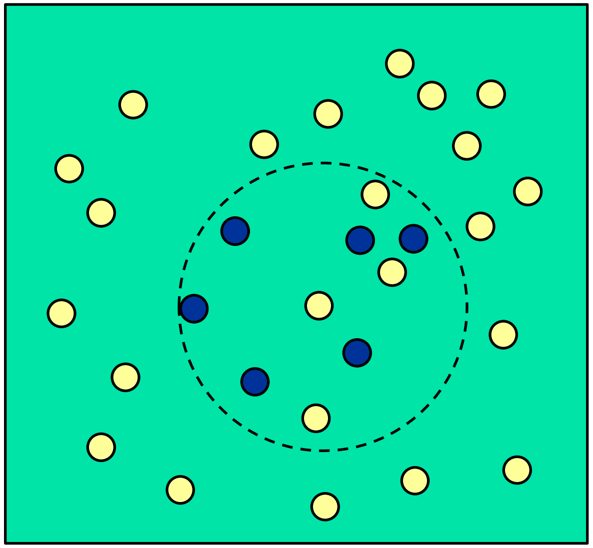
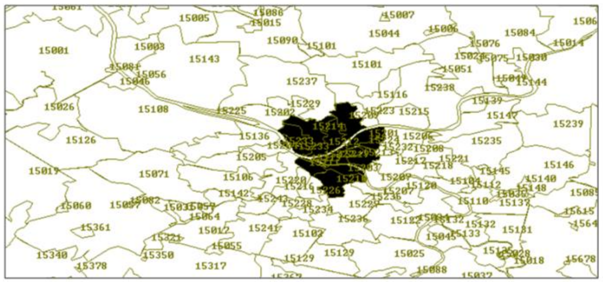


Larger p detects more compact clusters; smaller p detects more dispersed clusters.

The summed posterior probability $\sum_{S \subseteq S_{cn}} \Pr(S \mid D)$ can be efficiently computed for any p, $0 < p \leq 1$.

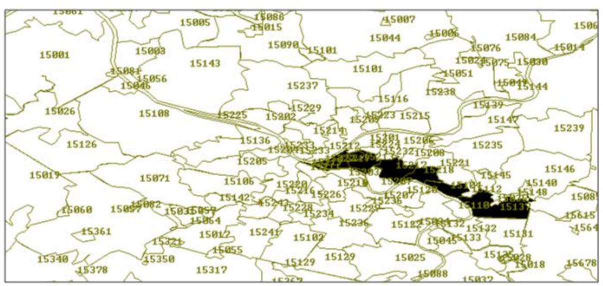
Generalized FSS

Optimization of the sparsity parameter p can substantially improve the detection performance of the generalized FSS approach.



<u>Compact cluster:</u>

Detection time minimized at p = 0.5; spatial accuracy maximized at p = 0.7.



<u>Highly elongated cluster:</u>

Detection time minimized at p = 0.1; spatial accuracy maximized at p = 0.2.

p = 0.2 improves detection time by 0.8 days and spatial accuracy by ~10%, as compared to p = 0.5.

Conclusions

FSS shares the essential advantages of MBSS: it can integrate information from **multiple data streams**, and can accurately distinguish between **multiple outbreak types**.

As compared to the original MBSS method, FSS substantially improves **accuracy** and **timeliness** of detection for elongated or irregular clusters, with similar performance for compact clusters.

While a naïve computation over the exponentially many subsets of the data is computationally infeasible, FSS can **efficiently** and **exactly** compute the posterior probability map.

Generalized FSS can further improve detection time and spatial accuracy through incorporation of a sparsity parameter.

We can also **learn** the prior distribution from a small amount of labeled training data, as in Makatchev and Neill (2008).

References

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