Making argument systems for outsourced computation practical (sometimes)

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By verified outsourced computation, we mean the following:

check whether $y = f(x)$, without computing $f(x)$

The motivation is 3\textsuperscript{rd} party computing: cloud, volunteers, etc.

We desire the following properties in the above exchange:

1. Unconditional, meaning no assumptions about the server
2. General-purpose, meaning not specialized to a particular $f$
3. Practical, or at least conceivably practical soon
Theory can supposedly help. Consider the theory of Probabilistically Checkable Proofs (PCPs).

Unfortunately, the constants and proof length are outrageous.

Using a naive PCP implementation, verifying multiplication of $400 \times 400$ matrices would take 500 trillion CPU years (seriously).
500 trillion is a big number.

For example, I can beat Michael Jordan in one-on-one basketball only one time out of 500 trillion.
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[ALMSS JACM98, AS JACM98]

Unfortunately, the constants and proof length are outrageous.

Using a naive PCP implementation, verifying multiplication of 400×400 matrices would take 500 trillion CPU years (seriously).
We have reduced the costs of a PCP-based argument system by Ishai et al. [CCC07] by 20 orders of magnitude, with proof.

We have implemented the refinements in a system, PEPPER, that is not ready for prime time but is practical in some cases.

Our conclusion is that PCPs are a potentially promising tool for building secure systems.
(1) The design of PEPPER

(2) Experimental results, limitations, and outlook
Pepper incorporates PCPs but not like this:

The proof is not drawn to scale: it is far too long to be transferred.

(Even the asymptotically short PCPs [BGHSV CCC05, BGHSV SIJC06, Dinur JACM07, BS SIJC08] have prohibitive constants.)
Instead of transferring the PCP …

… Pepper uses an efficient argument [Kilian CRYPTO 92,95]:

```
PCPQuery(q) {
    return <q, w>;
}

[IKO CCC07]
```
The server’s vector $w$ encodes an execution trace of $f(x)$.

What is in $w$?
(1) An entry for each wire; and
(2) An entry for the product of each pair of wires.
Instead of transferring the PCP ...

... Pepper uses an efficient argument [Kilian CRYPTO 92,95]:

```
PCPQuery(q) {
  return <q, w>;
}
```

[IKO CCC07]

This is still too costly (by a factor of $10^{22}$), but it is promising.
PEPPER incorporates four refinements to [IKO CCC07], with proof.

```
<table>
<thead>
<tr>
<th>client</th>
<th>server</th>
</tr>
</thead>
<tbody>
<tr>
<td>“f”, x</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td></td>
</tr>
<tr>
<td></td>
<td>commit request</td>
</tr>
<tr>
<td></td>
<td>commit response</td>
</tr>
<tr>
<td>query vectors: q_1, q_2, q_3, ...</td>
<td></td>
</tr>
<tr>
<td>response scalars: q_1 \cdot w, q_2 \cdot w, q_3 \cdot w, ...</td>
<td></td>
</tr>
</tbody>
</table>
```

ACCEPT / REJECT
This refinement works best for a restricted class of computations: straight-line, parallelizable, numerical.

Consider \( m \times m \) matrix multiplication as our computation \( f \):
- The Boolean circuit has \( O(m^3) \) gates \( \rightarrow w \) has \( O(m^6) \) entries
- The new representation has \( m^2 \) gates \( \rightarrow w \) has \( O(m^4) \) entries
“f”, x, y

commit request
commit response

query vectors: q₁, q₂, q₃, …

response scalars: q₁•w, q₂•w, q₃•w, …

ACCEPT/REJECT

client

server

w
We can sometimes exploit the structure of a computation.

Consider $m \times m$ matrix multiplication as our computation $f$:

This eliminates the server’s PCP-based overhead, and may apply to PCPs more broadly.
\textbf{ACCEPT/REJECT}

\textbf{commit request}

\textbf{query vectors: }q_1, q_2, q_3, \ldots

\textbf{response scalars: }q_1 \cdot w, q_2 \cdot w, q_3 \cdot w, \ldots
The client amortizes its overhead by reusing queries over multiple runs. Each run has the same $f$ but different input $x$. 

query vectors: $q_1, q_2, q_3, \ldots$

client

server

$w_1$

$w_2$

$w_3$
**PEPPER** generalizes the commitment primitive of Ishai et al. [CCC07].

With the new primitive, the client can issue multiple queries for the price of encrypting only a single query.
(1) The design of PEPPER

(2) Experimental results, limitations, and outlook
Consider amortized costs for multiplication of $400 \times 400$ matrices:

<table>
<thead>
<tr>
<th></th>
<th>Under the theory, naively applied</th>
<th>Under PEPPER</th>
</tr>
</thead>
<tbody>
<tr>
<td>client CPU time</td>
<td>$&gt;100$ trillion years</td>
<td>1.1 seconds</td>
</tr>
<tr>
<td>server CPU time</td>
<td>$&gt;100$ trillion years</td>
<td>1.6 hours</td>
</tr>
</tbody>
</table>

(assumes 2.4 Ghz CPU)

However, the batch size is large, so these numbers are not ideal.
PEPPER is not ready for prime time, for several reasons:

1. The client breaks even only for large batch sizes.
2. The server’s burden is too high, still.
3. The approach is plausible for only a class of computations.
We relate PEPPER to prior work in terms of our three goals.

1. General-purpose and practical; gives up unconditional
   - Replication ([Castro & Liskov TOCS02]), trusted hardware ([Chiesa & Tromer ICS10, SSW TRUST10]), auditing ([DJMM ICDCS04, HKD SOSP07, Kissner & Song ACNS04, MWR NDSS99])

2. Unconditional; gives up being general-purpose
   - [BGV CRYPTO11, Boneh & Freeman EUROCRYPT11, Golle & Mironov RSA01, Sion VLDB05, THHSY PET09, WRW INFOCOM11, Atallah & Friksen ASIACCS10, Freivalds MFCS79]
   - Toward practical Interactive Proofs [CMT ITCS12, GKR STOC08]

3. Unconditional and general-purpose; gives up practicality
   - Fully homomorphic encryption, secure multi-party computation [CKV CRYPTO10, GGP CRYPTO10, AIK ICALP10]
We have reduced the costs of a PCP-based argument system by Ishai et al. [CCC07] by 20 orders of magnitude, with proof. We have implemented the refinements in a system, PEPPER, that is not ready for prime time but is practical in some cases. Our conclusions are that PCPs are a potentially useful tool for real systems, and that the research area is promising.