Learning Theory and Algorithms for Second-Price Auctions with Reserve

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Auctions

- Standard method for buying or selling goods:
  - U.S. government: Treasury bills.
  - Christie’s or Sotheby’s: art.
  - eBay: everything, e.g., ‘honeymoon wife replacement’.
  - search engine companies: advertising rights.
Auctions

- Interaction between buyers and sellers:
  - game-theoretical analysis.
    - mechanism design.
    - study of properties.

- This talk:
  - learning theory analysis.
    - repeated auctions.
    - leveraging data.
Some Auction Types

- English auctions: interactive format; seller gradually increases the price until a single bidder is left.

- Dutch auctions (flowers in the Netherlands): interactive format; seller gradually decreases the price until some bidder accepts to pay.

- First-price sealed-bid auctions (e.g. NYC apartments): non-interactive; simultaneous bids, highest bidder wins and pays the value of his bid.
Second-Price Auctions

aka Vickrey auctions: e.g., eBay.

• bidders submit bids simultaneously.
• highest bidder wins and pays the value of the second-highest bid.
• truthful bidding is a dominated strategy.

(William Vickrey, 1961)
Truthfulness

Bidder $i$ with value $v_i$, other bids fixed.

- if $b_i > v_i$: change only if bidder wins and wasn’t before and second-highest bid is $b_j \in [v_i, b_i]$; payoff is $v_i - b_j \leq 0$.
- if $b_i < v_i$: change only if bidder loses and used to win. and second-highest bid $b_j \in [b_i, v_i]$; payoff was $v_i - b_j \geq 0$. 
SPA with Reserve

Second-price auctions with reserve: e.g., Ad Exchanges.
- seller announces a reserve price $r$ and,
- bidders submit bids simultaneously.
- winning bidder (if any) wins and pays the maximum of the value of the second-highest bid and $r$.
- truthful bidding is a dominated strategy.
Example

Suppose the seller’s value is 0 and there is a single bidder whose value is uniformly distributed over $[0, 1]$.

- no reserve price: item sold at value 0.
- reserve price: how should it be chosen?
  - probability $(1 - r)$ for bid being above $r$.
  - expected revenue $r(1 - r)$, thus $r = \frac{1}{2}$ is optimal.
Ad Exchanges
Ad Exchanges

- Significant fraction of the revenue of search engine and popular online sites:
  - Microsoft, Yahoo!, Google, OpenX, AppNexus.
  - Multi-billion dollar industry.

- Choice of reserve price:
  - main mechanism through which the auction revenue can be influenced.
  - if set too low, winner may end up paying too little; if set too high, the ad slot could be lost.

how can we select the reserve price to optimize revenue?
This Talk

- Learning formulation.
- Theoretical guarantees.
- Algorithms.
- Experimental results.
Previous ML Work

- Incentive compatible auctions (Balcan et al., 2008; Blum et al., 2004).
- Predicting bid landscapes (Cui et al., 2011).
- Revenue optimization for sponsored ads (Zhue et al., 2009; He et al., 2013; Devanur & Kakade, 2009).
- Bandit setting with no feature (Cesa-Bianchi et al., 2013; see also Ostrovsky & Schwarz, 2011).
- Strategic regret minimization (Amin et al., 2013; Munoz & MM, 2014).
Loss Function

- Auction revenue can be defined in terms of the pair of highest bids \( b = (b^{(1)}, b^{(2)}) \):

\[
\text{Rev}(r, b) = b^{(2)} \mathbb{1}_{r < b^{(2)}} + r \mathbb{1}_{b^{(2)} \leq r \leq b^{(1)}}.
\]

- Equivalently, loss define by

\[
L(r, b) = -\text{Rev}(r, b).
\]
Learning Formulation

- $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^N$: public information about auction (features).
- $\mathcal{B} \subseteq \mathbb{R}^2_+$: bid space.
- $H \subseteq \mathbb{R}^\mathcal{X}$: hypothesis set.
- $\mathcal{D}$ distribution over $\mathcal{X} \times \mathcal{B}$.

**Problem:** find $h \in H$ with small generalization error,

$$
\mathbb{E}_{(\mathbf{x}, \mathbf{b}) \sim \mathcal{D}}[L(h(\mathbf{x}), \mathbf{b})].
$$
Loss Function

- Properties:
  - discontinuous.
  - non-differentiable.
  - non-convex.

Can we derive guarantees for learning with this loss function?
Loss Decomposition
Generalization Bound

Theorem: let $M = \sup_{b \in B} b^{(1)}$ and let $H$ be a hypothesis set with pseudo-dimension $d = \text{Pdim}(H)$. Then, for any $\delta > 0$, with probability $1 - \delta$ over the choice of a sample $S$ of size $m$,

$$\mathcal{L}(h) \leq \mathcal{L}_S(h) + 2\mathcal{R}_m(H) + 2M \sqrt{\frac{2d \log \frac{em}{d}}{m}} + M \sqrt{\frac{\log \frac{1}{\delta}}{2m}}.$$ 

Can we design algorithms minimizing the right-hand side?
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No Feature Case

- **Problem**: find optimal reserve price,
  \[
  \min_{r \in \mathbb{R}} \sum_{i=1}^{n} L(r, b_i).
  \]

- **Algorithm**:
  - optimum one of highest bids.
  - naive in \( O(m^2) \).
  - sorting solution in \( O(m \log m) \).
Convex Surrogate Loss
No useful convex surrogate loss.

**Theorem:** Let $L_c : [0, M] \times [0, M] \rightarrow \mathbb{R}$ be a bounded function, convex with respect to its first argument. If $L_c$ is consistent with $(r, b) \mapsto -r1_{r \leq b}$, then $L_c(\cdot, b)$ is constant for every $b \in [0, M]$.

Which loss function should we use?
Continuous Surrogate Loss

loss function $L_\gamma$
Consistency Results

**Theorem:** let $M = \sup_{b \in B} b^{(1)}$ and let $H$ be a closed convex subset of a linear space of functions containing $0$. Then,

\[ \mathcal{L}(h^*) \leq \mathcal{L}(h^\gamma) \leq \mathcal{L}_\gamma(h^\gamma) + \gamma M. \]
Learning Guarantees

- **Theorem**: fix $\gamma \in (0, 1]$. Then, for any $\delta > 0$, with probability at least $1 - \delta$ over the choice of a sample $S$ of size $m$,

$$\mathcal{L}_\gamma(h) \leq \mathcal{L}_{\gamma,S}(h) + \frac{2\mathcal{R}_m(H)}{\gamma} + M\sqrt{\frac{\log \frac{1}{\delta}}{2m}}.$$
Algorithm

- Optimization problem: for fixed $\gamma \in (0, 1]$.

$$\min_{\|w\| \leq \Lambda} \sum_{i=1}^{m} L_\gamma(w \cdot x_i, b_i).$$

- difficulty: optimizing sum of non-convex functions.
- solution: DC-programming (Difference of Convex).
Difference of Convex Functions

\[ L_\gamma = u - v \]
Convex-concave procedure: replace $F(w) = f(w) - g(w)$ at iteration $(t + 1)$ with upper bound

$$\hat{F}(w) = f(w) - g(w_t) - \delta g(w_t) \cdot (w - w_t),$$

with $\delta g(w_t) \in \partial g(w_t)$. 

(Tao and Hoai, 1997; Yuille and Rangarajan, 2002)
Algorithm

SECONDPRICERESERVE()

1   \( w \leftarrow w_0 \)
2   \textbf{for} \( t \leftarrow 1 \textbf{ to } T \) \textbf{do}
3            \( v \leftarrow \text{DCA}(w_{t-1}) \)
4            \( u \leftarrow \frac{v}{\|v\|} \)
5   \( \eta^* \leftarrow \min_{0 \leq \eta \leq \Lambda} \sum_{u \cdot x_i > 0} L_\gamma(\eta u \cdot x_i, b_i) \)
6   \( w_t \leftarrow \eta^* v \)
7   \textbf{return} \( w \)
Line Search

- Observation: $L_{\gamma}$ is positive homogenous, for all $\eta > 0$,
  
  \[
  L_{\gamma}(\eta r, \eta b) = \eta L_{\gamma}(r, b).
  \]

- Consequence: line search equivalent to no-feature minimization algorithm; for $\mathbf{w}_0 \mathbf{x}_i > 0$
  
  \[
  \sum_{i=1}^{m} L_{\gamma}(\eta \mathbf{w}_0^\top \mathbf{x}_i, b_i) = \sum_{i=1}^{m} (\mathbf{w}_0^\top \mathbf{x}_i) L_{\gamma} \left( \eta, \frac{b_i}{\mathbf{w}_0^\top \mathbf{x}_i} \right).
  \]
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Experimental Results
Distribution of Reserve Prices
eBay Sport-Card Data Set

Data: [http://cims.nyu.edu/~munoz/data](http://cims.nyu.edu/~munoz/data).
Conclusion

• Theory, algorithms, and experiments for second-price auctions with reserve.
  • scaling up DC algorithm.
  • study of dependencies.
  • effect of using revenue optimization algorithm.
  • better initialization.

• Learning and auctions:
  • many other scenarios and types of auctions.
  • Example: Generalized Second-Price auctions (GSPs).