DESIGN OF WAVEGUIDE, CAVITY AND A COUPLING ARRANGEMENT USING CENTRED CIRCULAR APERTURE TO OPERATE AT A FIXED FREQUENCY OF 8 GHz

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DESIGN PREPARED BY

BHUBANESWAR MISHRA

DEPARTMENT OF ELECTRONICS AND ELECTRICAL

COMMUNICATION ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY

KHARAGPUR - 721302.

: C O N T E N T S :

ELECTRONIC AND TELECOMMUNICATION ENGINEERING DISCIPLINE

STATEMENT OF THE PROBLEM :

It is required to design a cylindrical metal waveguide to resonate at 8 GHz. The cavity is to be coupled to a cylindrical waveguide carrying the dominant TE_{11} mode by means of a centred circular aperture.

Calculate and specify all dimensions of cavity, cylindrical waveguide and the centred circular aperture. What is the mode (or modes) in which the cavity will resonate? Calculate the Q of the cavity, if the cavity is made of

1. BRASS
2. SILVER COATED BRASS

INTRODUCTION :

Transmission line (feeders)and oscillatory systems constitute an essential part of almost any electronic circuits. However, the linear dimensions of oscillatory circuits at microwave frequencies become comparable to the operating wavelength so that the concept of an oscillatory system with lumped parameters loses here any physical meaning.

Hence at frequencies above 1000 MC (1 GHz) it isessential to use metallic enclosures or cavities, in which a dielectric space is enclosed by a conducting material.

The history of these resonators goes back many years. In 1893 J.J. Thompson derived an expression for resonant frequencies of the transverse electric modes in cylinder. In 1897 Lord Rayleigh published a paper dealing with TE_{01n} mode in cavity which gives the highest Q for smallest volume.

- These metallic cavities are mainly of any of the three shapes, a) Rectangular, b) Cylindrical, c) Spherical. Since the power loss in a cavity is proportional to the surface area, whereas the power stored depends on volume, and since the Q of a cavity depends on the ratio of volume to area, the sphere with minimum surface area and highest volume seems preferable. However, sphere has other disadvantages,such as, the resonant frequency cannot be varied easily, problem of mode degeneracy and difficulty in mode excitation. For the above reasons, the cylindrical cavity finds maximum use.

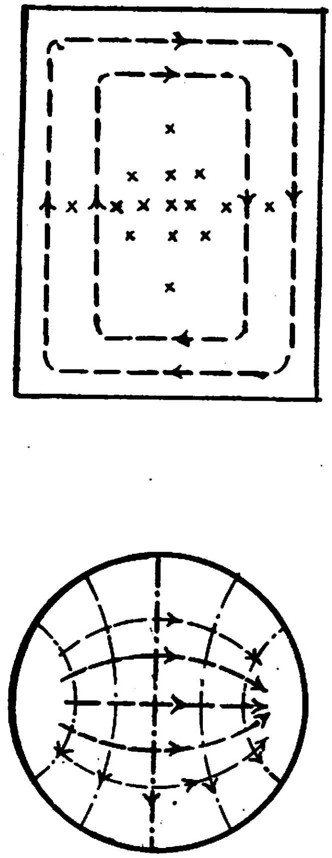
The cavity can have a variable resonant frequency (variation can be achieved by means of a plunger) or it can have a fixed resonant frequency. Both types are of equal importance in microwave applications.

The microwave cavities find wide applications in microwave circuits, such as, oscillator, tuned amplifier, frequency filters and many other measurement devices. The importance of these resonators in above applications is due to their ability to select a particular frequency very sharply.

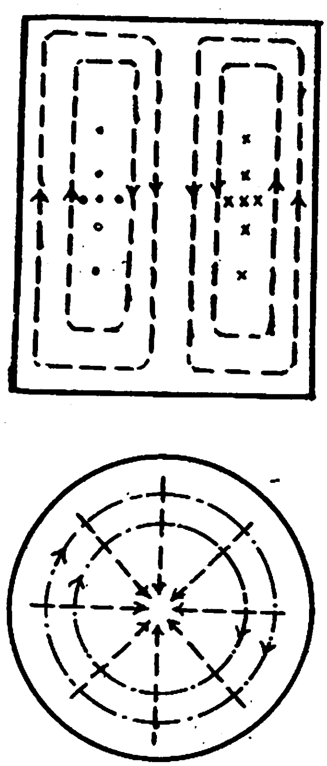
The cylindrical waveguide coupled to a cavity system can be used for frequency control of microwave oscillators as in Pound Oscillator. In another important use, the cavity coupled to a waveguide can be used as a radiator of high power with low bandwidth.

In all these applications it is always preferred that the cavity has a high Q, and the coupling to the external system (exciter or extractor) be critical.

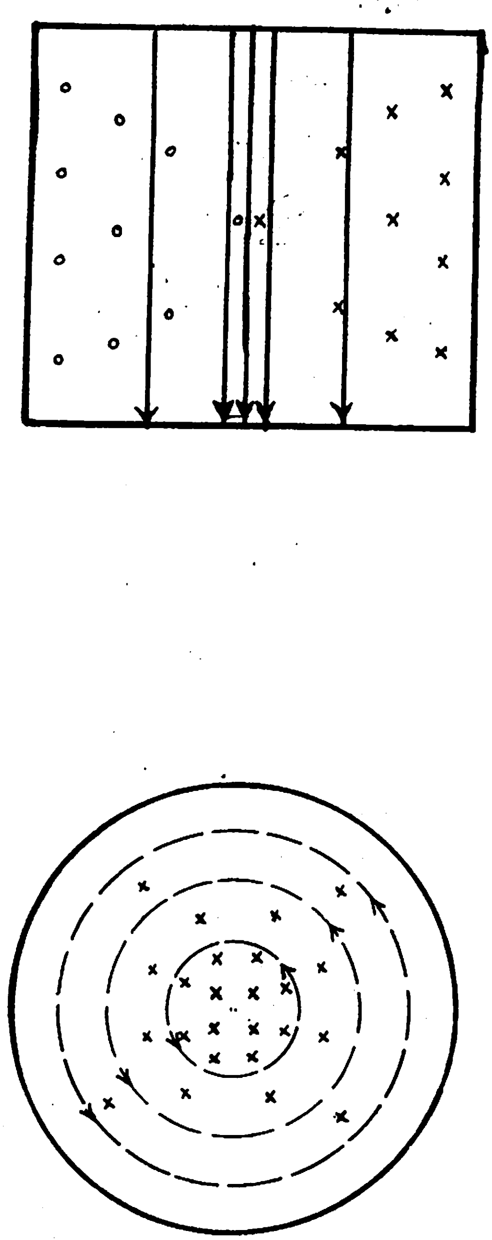
Depending on the field distribution in a cavity, the cavity can sustain a number of resonant frequencies, each frequency corresponding to certain resonant mode. Each reasonant mode is characterized by a certain field distribution and will have a certain value of Quality Factor, Q and a resonance frequency. Hence depending on the use a particular mode is to be chosen.



A. Field Configurations for TE_{111} mode.



B. Field Configurations for TE_{011} mode



C. FIELD CONFIGURATIONS FOR TM_{010} MODE.

Hence, now it is required that an arrangement for exciting this cavity be made. The coupling of the cavity is a waveguide depending on the basis :

a) The mode to be excited in the cavity;

b) The coefficient of coupling required.

For our purpose, the arrangement of critical coupling is chosen.

The coupling of waveguide to cavity can be done by probes or loops, or aperture. However, probes and loops are suitable for transmission line, whereas aperture coupling is the best for waveguide.

The magnetic and electric field configurations of a cylindrical cavity for different modes have been shown in the accompanying figure.

From the examination of these figures, we find that the magnetic field lines for TE_{011} mode is perpendicular to the cavity wall, this resulting in very small copper loss on the wall. As a result, it can be inferred that the Q of such cavities will be the highest and most suitable for the purpose. As a result, the design problem given reduces to the following :

a) Design of a cylindrical waveguide carrying 8 GHz, that will excite a cavity.

b) Design of a cavity that will resonate a 8 GHz and supporting the TE_{011} mode for high Q.

c) Location and dimension of an aperture that will excite the TE_{011} mode while suppressing the unwanted mode and the critical coupling will be achieved.

The different aspects of this design problem have been discussed in chapters which follow this.

Chapter 2 begins with the derivations of the field equations required in the design of a waveguide. Different considerations, such as, flatness of attenuation factor at the operating frequency, mode separation, power handling capacity of the guide, breakdown voltage, etc., have been discussed in reference to the design of the guide. A similar discussion for cavity design along with required derivations for field equations are given. The cavity mode has been selected based on minimizing the extraneous mode and realization of high Q. This is followed by a discussion of coupling arrangements possible from the consideration of cavity and waveguide field distribution. A suitable coupling arrangement is chosen based on the critical coupling and suppression of unwanted mode. Required expression for dimension of aperture has been derived. This is accompanied by required diagrams and print out of the computer program with the results, used in the calculation.

In Chapter 3 all the calculations based on the derivations given in the previous chapter are made. The dimensions of waveguide, cavity and the aperture are calculated. The loaded and unloaded Q of the cavity are calculated for cavities made of a) Brass (90% copper as well as 70% copper) and b) silver coated brass.

In Chapter 4 an alternative scheme has been considered. Though this scheme suffers from serious disadvantages like very low Q and inadequate suppression of unwanted mode, it has an edge over the previous scheme, in that it is simple to fabricate and the iris

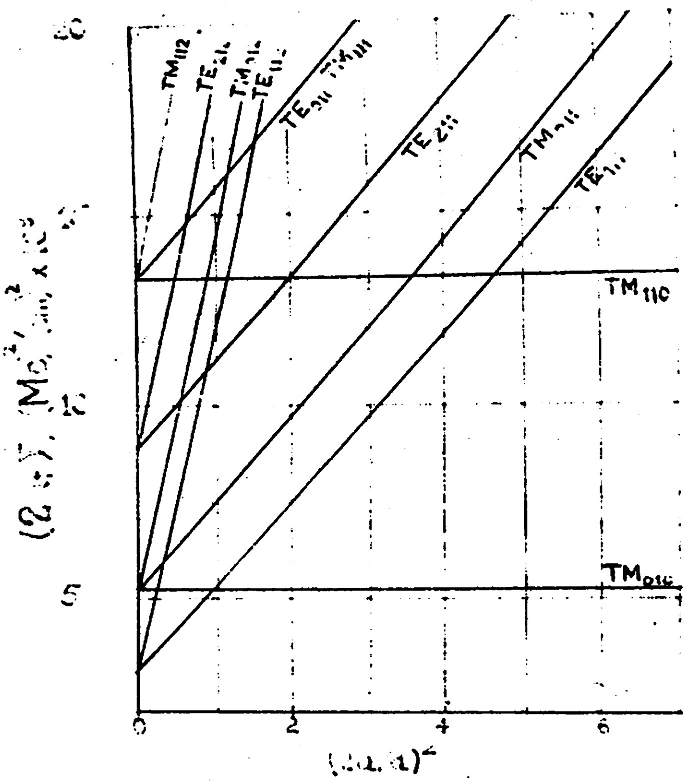


FIG. 1. MODE CHART

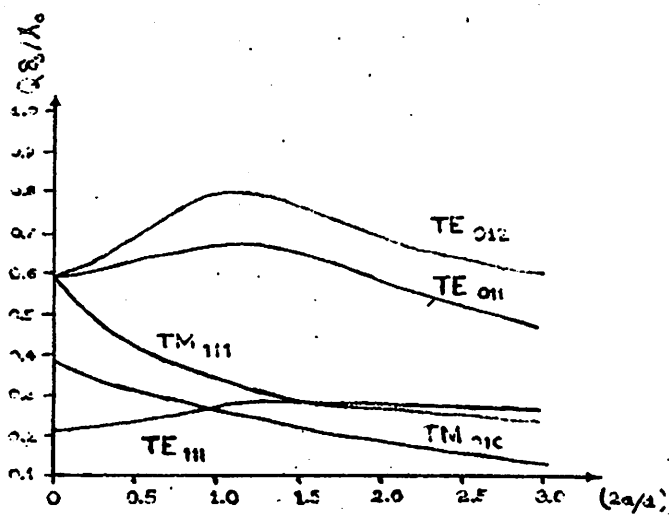


FIG. 2. Q FOR DIFFERENT MODES

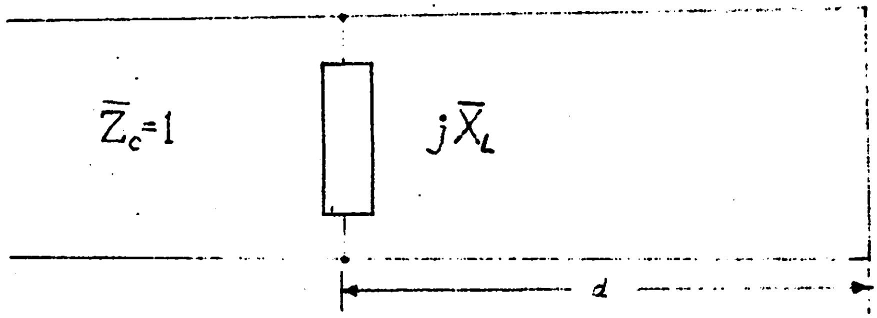


FIG. 3. EQUIVALENT TRANSMISSION-LINE CIRCUIT (SCHEME 1).

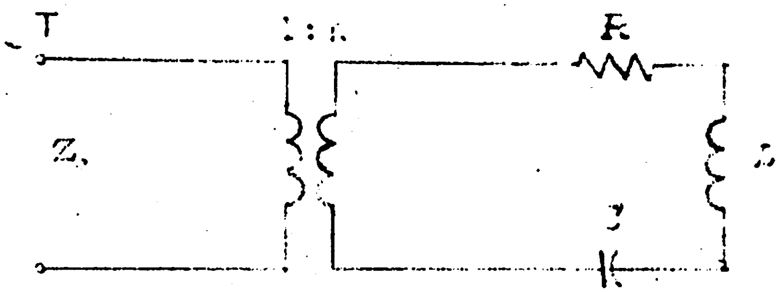


FIG. 4. EQUIVALENT CIRCUIT (SCHEME 2).

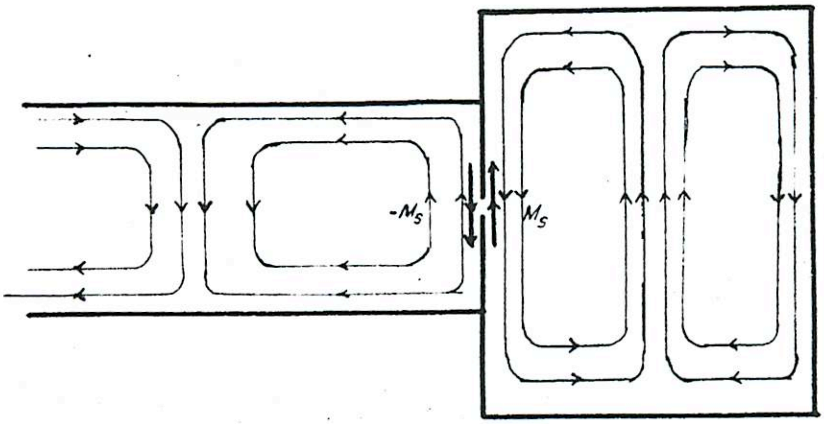


FIGURE 5 ALLIGNMENT OF FIELD PATTERNS IN TE_{11} MODE CYLINDRICAL WAVEGUIDE WITH TE_{011} MODE CYLINDRICAL CAVITY.

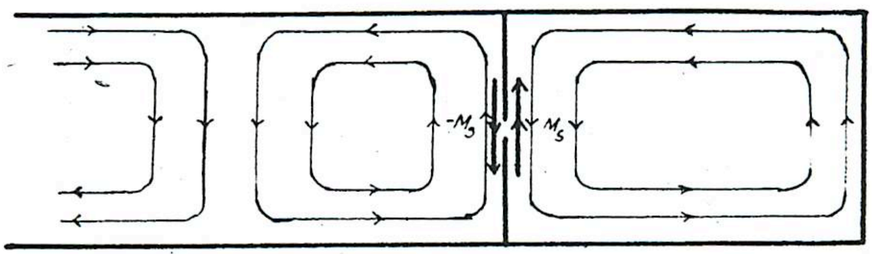


FIGURE.6. ALLIGNMENT OF FIELD PATTERNS IN TE_{11} MODE CYLINDRICAL WAVEGUIDE WITH TE_{111} MODE CYLINDRICAL CAVITY.

lies on the centre of the transverse wall of both waveguide as well as cavity. The required field expression and calculations are given in brief.

In Chapter 5, the procedure to be adopted while fabricating the system is described. Other additional features, such as, effect of temperature and humidity on cavity Q are described.

In the event of inadequate suppression of unwanted mode by the coupling arrangement, an arrangement for mode suppression by and plate gaps can be adopted. A detailed description of this will be found here. Some other arrangements for increasing Q have been discussed in brief.

We conclude this design with a brief overview of the design techniques and highlighting only the most important considerations.

This is followed by an appendix containing tables required for the design and the list of books, journals, handbooks and other technical reports used for the report.

BRIEF THEORY AND DERIVATIONS USED IN THE DESIGN :

2.1. WAVEGUIDE

2.1.1. THE SOLUTION OF THE ELECTROMAGNETIC EQUATIONS FOR GUIDES OF CIRCULAR CROSS SECTION :

For cylindrical waveguide with circular cross section, [1] the field and wave equations are expressed in cylindrical coordinate (ρ, ϕ, Z), so that boundary conditions can be applied easily.

In cylindrical coordinates in a nonconducting source-free region (assuming the variation in x-direction to be given by $e^{-\Gamma Z}$, where   is the constant of propagation), Maxwell's Equations are

$$
\left.
\begin{aligned}
\frac{\partial H_z}{\rho\,\partial\phi} + \bar{\gamma} H_\phi &= j\omega\epsilon E_\rho \\
\frac{\partial E_z}{\rho\,\partial\phi} + \bar{\gamma} E_\phi &= -j\omega\mu H_\rho \\
-\bar{\gamma} H_\rho - \frac{\partial H_z}{\partial\rho} &= j\omega\epsilon E_\phi \\
-\bar{\gamma} E_\rho - \frac{\partial E_z}{\partial\rho} &= -j\omega\mu H_\phi \\
\frac{1}{\rho}\left(\frac{\partial(\rho H_\phi)}{\partial\rho} - \frac{\partial H_\phi}{\partial\phi}\right) &= j\omega\epsilon E_z \\
\frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial\rho} - \frac{\partial E_\rho}{\partial\phi}\right) &= -j\omega\mu H_z
\end{aligned}
\right\} \qquad (2.1.1)
$$

The wave equation in cylindrical coordinates for H_z is obtained from (2.1.1)

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho^2}\,\frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} + \frac{1}{\rho}\,\frac{\partial H_z}{\partial \rho} = -\omega^2 \mu \epsilon H_z \qquad \ldots (2.1.2)$$

By separation of variables and after identifying the Bessel equation, we obtain solution for H_z as

$$H_z = J_n(\rho h)\,(C_n \cos n\phi + D_n \sin n\phi)\, e^{-\bar{\gamma} z} \qquad \ldots (2.1.3)$$

The boundary condition requires that

$$\left.\frac{\partial H_z}{\partial \rho}\right|_{\rho = a} = 0 \qquad \left.\frac{\partial J_n(\rho h)}{\partial \rho}\right|_{\rho = a} = J_n'(ah) = 0 \qquad \ldots (2.1.4)$$

$$\therefore \quad h = \frac{p'_{nm}}{a}$$

Hence for TE_{11} (dominant mode) in a cylindrical waveguide, the field equations are as given below [3]

$$\left.\begin{aligned}
H_z &= J_1\left(\frac{p'_{11}}{a}\right) e^{-j\beta_{11} z} \begin{Bmatrix} \cos\phi \\ \sin\phi \end{Bmatrix} \\
E_z &= 0 \\
H &= -\frac{j\beta_{11}\, p'_{11}}{a k_{c_{11}}^2}\, J_1'\left(\frac{p'_{11}\rho}{a}\right) e^{-j\beta_{11} z} \begin{Bmatrix} \cos\phi \\ \sin\phi \end{Bmatrix} \\
H_\phi &= -\frac{j\beta_{11}}{\rho k_{c_{11}}^2}\, J_1\left(\frac{p'_{11}\rho}{a}\right) e^{-j\beta_{11} z} \begin{Bmatrix} -\sin\phi \\ \cos\phi \end{Bmatrix} \\
E &= Z_{h,11}\, H_\phi \\
E_\phi &= -Z_{h,11}\, H_\rho
\end{aligned}\right\} \qquad \ldots (2.1.5)$$

The other factors for the cylindrical waveguide transmitting TE_{11} mode are

propagation constant for TE_{11} mode

$$\beta_{11} = \left[k_0^2 - \left(\frac{p'_{11}}{a}\right)^2 \right]^{1/2} \qquad \ldots (2.1.6)$$

wave impedance for TE_{11} mode

$$Z_{h,11} = \frac{k_0}{\beta_{11}} z_e \qquad \ldots (2.1.7)$$

cut-off wave number for TE_{11} mode is

$$K_{c,11} = \frac{p'_{11}}{a} \qquad \ldots (2.1.8)$$

the cut-off wavelength for TE_{11} mode is

$$\lambda_{c,11} = \frac{2\pi a}{p'_{11}} \qquad \ldots (2.1.9)$$

where $p'_{11} = 1.84$ and $\lambda_c = 3.41a$

and $(f_c)_{11} = 0.293/_a \sqrt{(/\mu_1 \epsilon_1)}$ $\qquad \ldots (2.1.10)$

2.1.2. DESIGN CONSIDERATIONS FOR CYLINDRICAL WAVEGUIDE :

The dimensions of a waveguide (here the diameter of the cylindrical waveguide) are chosen mainly on two basis :

a) The dimensions are so chosen that the operating frequency is about 20% higher than cut-off frequency, where the attenuation and phase shift factor with respect to frequency is more or less flat and the medium acts like an almost nondispersive guide.

b) There is enough mode separation between the dominant mode in the guide and the next higher order mode possible. This is essential since a degeneracy in mode results in complexities associated with coupling signal energy into and out of a waveguides when more than one mode propagates.

We find that TE_{11} is the dominant mode and the next higher order mode is TM_{01}.

The cut-off wavelength for TM_{01} mode is

$$(\lambda_c)_{TM_{01}} = \left(\frac{2\pi a}{P_{01}}\right) = \frac{6.28a}{2.405} = 2.611a \qquad \ldots \quad (2.1.11)$$

Other consideration in the choice of waveguide dimensions are

(i) the dimension for which attenuation factor is minimum represents a suitable choice

$$\alpha = \frac{R_m}{aZ_0}\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{-1/2}\left[\left(\frac{f_c}{f}\right)^2 + \frac{1}{p'^2_{11} - 1}\right] \quad \ldots \quad (2.1.12)$$

However, the minimum occurs at a point where both TE_{11} and TM_{01} exist together. Hence this choice is not suitable.

(ii) The other choices are maximum power handling capacity and the breakdown voltage that can be sustained. From the expression for power we find that the power handling capacity improves as the operating frequency increases.

From the expression for the electric field we find that with higher dimensions the possibility of breakdown decreases.

The calculations based on these considerations are given in Section 3.1.

2.2. CAVITY

2.2.1. SELECTION OF CYLINDRICAL CAVITY TO BE COUPLED TO THE CYLINDRICAL WAVEGUIDE

The mode of the cavity is chosen based mainly on the following two basis :

a) The cavity of minimum volume for given Q, because the number of resonant frequencies possible below a certain frequency is proportional volume as will be shown below.

b) A cavity having a minimum of extraneous responses of the type difficult to suppress.

While designing a cavity to be coupled to a waveguide the design is based on the selection of a mode which will give the highest Q per volume, while suppressing the other unwanted modes. The later consideration is very important and is taken into consideration while designing the coupling arrangement.

Every cavity resonator regardless of its shape has a series of resonant frequencies and infinity in number and more closely spaced as the frequency increases. The total number N having a resonant frequency less than f is given by [13]

$$N = \frac{8\pi}{3c^3} V f^3 \qquad \ldots (2.2.1)$$

where V = Volume of cavity (in cubic meter)

c = Velocity of electromagnetic wave (m/sec.)

f = frequency (cycles per second)

Now we consider which mode has the highest Q for a given volume. It is desirable to keep the volume minimum since the total number of resonance is a function of volume Analysis of the problem leads to the conclusion that operation in TE_{01n} mode gives the smallest volume for an assigned Q.

From the considerations given above and considerations of suppression of mode associated we chose TE_{011} mode for the cylindrical cavity. Since the TM_{111} will always be present with TE_{011}, we couple the cavity to waveguide so that there can be exchange of energy while TM_{111} is suppressed.

The coupling scheme with corresponding field diagrams are shown in Fig.7 and Fig.5 respectively. The aperture is on the centre of the transverse wall of the waveguide, connecting it to the cylindrical wall of the cavity.

2.2.2. THE FIELD EQUATION AND UNLOADED Q OF THE CAVITY :

We find out the field equations for a cylindrical cavity of length d and radius a_c. This is a section of circular waveguide with short circuiting plates at each end.

The fields in the cylindrical cavity may be determined from the corresponding waveguide solutions. Using the appropriate boundary conditions near the short circuiting walls, we obtain the field equations for TE_{011} mode.

$$\left.\begin{aligned} H_z &= -2_j H_1 J_0 \left(\frac{p'_{01}}{a_c}\rho\right) \sin\left(\frac{\pi z}{d}\right) \\ H &= \frac{-2_j}{(p'_{01}/a_c)}\left[\frac{\pi}{d} H_1 J'_0 \left(\frac{p'_{01}}{a_c}\rho\right) \cos\left(\frac{\pi z}{d}\right)\right] \\ E_\phi &= \frac{2\omega\mu_0}{(p'_{01}/a_c)} H_1 J'_0 \left(\frac{p'_{01}}{a_c}\rho\right) \sin\left(\frac{\pi z}{d}\right) \end{aligned}\right\} \quad \ldots (2.2.2)$$

To find Q of the TE_{011} mode, the time average energy W stored in the cavity and power loss in the walls must be calculated

$$\left.\begin{aligned} W &= 2W_e = \frac{\epsilon_0}{2}\int_0^{a_c}\int_0^{2}\int_0^{d} \left(|E_r|^2 + |E_\phi|^2\right) r\,d\phi\,dr\,dz \\ P_l &= \frac{R_m}{2}\int_{walls} |H_{tan}|^2\,ds \end{aligned}\right\} \quad \ldots (2.2.3)$$

Substituting these values we find,

Mode Shape Factor = $Q\,\delta_s/\lambda_0$ =

$$\frac{\left[(p'_{01})^2 + \frac{\pi^2}{4}\left(\frac{2a_c}{d}\right)^2\right]^{3/2}}{2\pi\left[(p'_{01})^2 + \frac{\pi^2}{4}\left(\frac{2a_c}{d}\right)^3\right]} \quad \ldots (2.2.4)$$

However, we find that the Mode Shape Factor is a complex function the ratio $(2a_{c/d})$. If we call Mode Shape Factor, M and the dimension ratio of diameter to length, r, then we find

$$M = \frac{\left[(p'_{01})^2 + \frac{\pi^2}{4} r^2\right]^{3/2}}{2\pi\left[(p'_{01})^2 + \frac{\pi^2}{4} r^3\right]} \qquad \ldots \quad (2.2.5)$$

Now, by differenting M with respect to r, we can maximize it, i.e., find out the r for which M takes a maximum value

$$\frac{dM}{dr} = (p'_{01})^2 \left[1 - r\right] = 0$$

$$r = 1$$

$$2a_c = d \qquad \ldots \quad (2.2.6)$$

However, from the graphs given in Fig.2, we find that the M.S. Factor is considerably flat near the maximum and for the design we can choose

$$2a_c \simeq d \qquad \ldots \quad (2.2.7)$$

2.2.3. DESIGN CONSIDERATIONS FOR CYLINDRICAL CAVITY :

The design of the cavity dimensions is facilitated by/the using mode chart and defining an operating reactangular area on it. The choice of this area is based on the suppression of the following modes :-

a) Interference mode, i.e., any non diagonal modein this area.

b) Most care is taken for suppression of self interference mode which, as will be shown, leads to TE_{011} mode.

c) Crossing modes in the area are avoided at all cost, since it degrades the Q seriously.

The dimensions for the cavity are chosen so that within the bandwidth of the cavity, the interfering and crossing modes are avoided. This choice can be easily and very efficiently done by using a Mode Chart as shown in Fig.1.

The Mode Charts are constructed based on the following relations between the resonant frequency and diameter to length ratio :

$$f_{nml} = \frac{k_{nml}c}{2\pi} = \left[\left(\frac{p_{nm}}{a_c}\right)^2+\left(\frac{l\pi}{d}\right)^2\right]^{1/2}\frac{c}{2\pi} \quad \ldots (2.2.8)$$

$$(2a_c f_{nml})^2 = \left(\frac{cp_{nm}}{\pi}\right)^2+\left(\frac{cl}{2}\right)^2\left(\frac{2a_c}{d}\right)^2$$

Using the equation 2.2.8, the line corresponding to TE_{011} mode in the mode chart will be given by the following equation :

$$(2a_c f_{011})^2 = \left(\frac{cp'_{01}}{\pi}\right)^2+\left(\frac{c}{2}\right)^2\left(\frac{2a_c}{d}\right)^2 \quad \ldots (2.2.9)$$

Since the cavity to be designed is a tuned cavity with a very high Q, the bandwidth of this cavity will be very low.

Now for this small frequency variation, we can choose a rectangle on the mode chart of which TE_{011} forms the diagonal.

Any non-diagonal mode within this rectangular area will be an extraneous mode and can lead to confusion and ambiguity. Since the size of this rectangle is very small, the interfering modes (i.e., those which do not cross the desired mode) does not cause much problem.

For TE_{011} mode the self-interference is also not a majot problem, since, for TE_{01n} mode

$$\frac{f_2}{f_1} = \frac{n+1}{n} \qquad \ldots \quad (2.2.10)$$

and this ratio for TE_{011} is equal to 2.

The cross-mode (i.e., an undesired mode that crosses the main mode within the rectangle) can become a major source of problem if the diameter to length ratio is not chosen properly. Because in the immediate region of the crossing point, the cavity is simultaneously resonant in both the modes and violent interaction effect which may seriously degrade the cavity Q frequently occur at such crossings.

From the above considerations and examination of the Mode Chart shows that a value of $(2a_{c/d})^2 \approx 2$ satisfies all the requirements.

The detailed calculations are given for the cavity design in Section 3.2.

2.3. APERTURE :

2.3.1. CHOICE OF THE COUPLING ARRANGEMENT :

Choice of the coupling arrangement is made on the following basis :-

a) The coupling by electrical field or magnetic field is chosen on the basis of mode to be excited and hence probe or aperture (in place of aperture loop can be used).

b) The location of the aperture is chosen on two basis :

(i) The aperture is located so that it couples to the maximum magnetic field.

(ii) The coupling should be such that it suppresses the unwanted mode while exciting the required mode.

To be useful the cavity must be coupled to external circuits. The problem here is to get the correct coupling to the main mode and as little coupling as possible to all other modes. Since the electric field is zero everywhere at the boundary surface of the cavity for TE_{011} mode, coupling to it must be magnetic. This may be done by a loop or an orifice connecting the cavity to the waveguide.

Here we have chosen a coupling by means of a circular aperture on the centre of the transverse wall of the waveguide connecting to the cavity on the cylindrical wall.

The location for the maximum coupling to the main mode is on the side of the cavity, equal to a greater wavelength from the end (i.e., at the middle of the side wall of the cavity), or on the end about halfway (48%) out from the centre of the edge. Correct orientation of waveguide is obtained when the magnetic field is parallel.

However, the coupling arrangement can be used in a meaningful manner, allowing us to suppress the unwanted mode.

From the mode chart we find that the wanted TE_{011} mode is paired with unwanted TM_{111} mode. Since TM mode has Q substantially less than that of the TE, the fact that makes the realization of higher Q difficult, it is required to choose the coupling which will suppress the unwanted TM mode. This is possible because the type and location of the coupling means can be used to discriminate between wanted and unwanted means. Since TM mode has $H_z = 0$, orifice coupling to the main mode at side wall will not cuple to any TM modes.

Based on these considerations we chose the coupling scheme shown in Fig.7 with the field configuration shown in Fig.5.

2.3.2. THE CHOICE OF THE APERTURE RADIUS :

The radius of the aperture is chosen such that the coupling is critical. This critical coupling is necessary since this can find application in measurement devices where a high dip is required for proper measurement.

The derivations that follow for this aperture coupling is based on the general problem of coupling a cavity of any shape to a waveguide through an aperture as discussed in "Time Harmonic Electromagnetic Fields" : R.F. Harrington (pp.436-438) [8].

An equivalent circuit for this coupling arrangement is found out in the vicinity of resonance. The elements representing the result of evanescent modes are neglected since it complicates the derivations.

The assumptions used are -

a) Orifice is in a wall of negligible thickness;

b) Its diameter is small compared to the wavelength;

c) The orifice is not near any surface discontinuity;

d) Waveguide propagates only the dominant mode and is properly terminated.

For a circular waveguide, we have

$$\psi^{e}_{np} = \frac{\sqrt{\epsilon_n}}{\sqrt{\pi\left[(p'_{np})^2 - n^2\right]}} \; \frac{J_n(x'_{np}\rho/a)}{J_n(x'_{np})} \begin{cases} \sin n\phi \\ \cos n\phi \end{cases} \quad \ldots (2.3.1)$$

For dominant TE_{11} mode

$$\psi^{e}_{11} = \frac{1}{\sqrt{\pi\left[(p'_{11})^2 - 1\right]}} \; \frac{J_1(p'_{11}\rho/a)}{J_1(p'_{11})} \sin\phi \quad \ldots (2.3.2)$$

Mode Function for TE_{11} is

$$e^{e} = K\left[U_z \times \nabla_t \psi^{e}_{11}\right]$$

$$= -\hat{U}_\rho \frac{K}{\rho\sqrt{\pi\left[(p'_{11})^2 - 1\right]}} \; \frac{J_1(p'_{11}\rho/a)}{J_1(p'_{11})} \cos\phi$$

$$+ \hat{U}_\phi \frac{K}{\sqrt{\pi\left[(p'_{11})^2 - 1\right]}} \; \frac{J'_1(p'_{11}\rho/a)}{J_1(p'_{11})} \left(\frac{p'_{11}}{a}\right) \sin\phi \quad \ldots (2.3.3)$$

Normalizing the mode vector, we obtain

$$\int {e^{e}}^2 d\tau = 1 \quad \ldots (2.3.4)$$

where the integration is overthe guide cross section.

$$K = \frac{J_1(p'_{11})\sqrt{(p'_{11})^2 - 1}}{\sqrt{I_1 + I_2}} \qquad \ldots (2.3.5)$$

$$= \sqrt{2}$$

$$\text{where } I_1 = \int_0^{p'_{11}} \theta\, J_1'^2(\theta)\, d\theta$$

$$I_2 = \int_0^{p'_{11}} \frac{J_1^2(\theta)}{\theta}\, d\theta$$

We assume an aperture field as

$$E_t^a = -\hat{U}_\rho \frac{1}{\left(\frac{p'_{11}\rho}{a}\right)} J_1\left(\frac{p'_{11}\rho}{a}\right) \operatorname{Cos}\phi + \hat{U}_\phi J_1'\left(\frac{p'_{11}\rho}{a}\right) \operatorname{Sin}\phi \qquad \ldots (2.3.6)$$

Hence the waveguide dominant mode voltage is

$$V_o = \left(\frac{a}{p'_{11}}\right) J_1(p'_{11}) \sqrt{\frac{\pi\left[(p'_{11})^2 - 1\right]}{2}} \qquad \ldots (2.3.7)$$

Now the waveguide dominant mode voltage that would be excited if the waveguide were of the same radius as the aperture

$$V = \left(\frac{r_o}{p'_{11}}\right) J_1(p'_{11}) \sqrt{\frac{\pi\left[(p'_{11})^2 - 1\right]}{2}} \qquad \ldots (2.3.8)$$

where r_o is the radius of the aperture.

Hence the turns ratio of the ideal transformer is given by the expression

$$n^2 = \left(\frac{V}{V_o}\right)^2 = \left(\frac{r_o}{a}\right)^2 \qquad \ldots (2.3.9)$$

For a small aperture we assume the field uniform on the cavity side and near the wall

$$H_o = \hat{U}_\rho \, H \, J_o(p'_{o1}) \, \mathrm{Sin}\phi + \hat{U}_\phi \, H \, J_o(p'_{o1}) \, \mathrm{Cos}\phi \qquad \ldots (2.3.10)$$

By normalizing power we get

$$H = \frac{1}{\sqrt{\mu \pi d \, I_3} \left(\frac{a_c}{p'_{o1}}\right) \left[1 + \left(\frac{\pi}{p'_{o1}} \frac{a_c}{d}\right)^2\right]^{1/2}} \qquad \ldots (2.3.11)$$

and $I_3 = \int_o^{p'_{o1}} \theta \; J_o'^2(\theta) \, d\theta$

Now

$$b_o = \iint_{\text{aperture}} E_t^a \times H_o \, \rho \, d\rho \, d\phi \qquad \ldots (2.3.12)$$

$$= H \, \pi \, J_o(p'_{o1}) \left(\frac{r_o}{p'_{11}}\right)^2 \left[I_4 + I_5\right]$$

Where $I_4 = \int_o^{p'_{11}} J_1(\theta) \, d\theta$

$$I_5 = \int_o^{p'_{11}} \theta \, J_1'(\theta) \, d\theta$$

Using numerical integration on a computer (Simpson's method) we find

$$I_3 = 1.190823$$

$$I_4 = 0.691000$$

$$I_5 = 0.388314$$

Using the above values we obtain

$$\left(\frac{V}{b_o}\right)^2 = 0.6347 \left\{ \mu d \, \frac{a_c^2}{r_o} \left[1 + \left(\frac{\pi}{p'_{o1}} \, \frac{a_c}{d} \right)^2 \right] \right\} \quad \ldots \quad (2.3.13)$$

For critical coupling the load referred to primary at resonant frequency should be equal to the characteristic admittance of the dominant mode.

$$\therefore \frac{n^2}{R} = Y_o \quad \ldots \quad (2.3.14)$$

Since L is given by

$$L = (V/b_o)^2 \quad \ldots \quad (2.3.15)$$

R in the above expression is given by

$$\frac{1}{R} = \frac{Q}{\omega_o} \left(\frac{b_o}{V}\right)^2 \quad \ldots \quad (2.3.16)$$

$$\therefore \frac{n^2 Q}{\omega_o} \left(\frac{b_o}{V}\right)^2 = Y_o \quad \ldots \quad (2.3.17)$$

The detailed calculation of the aperture radius using the above equations is given in Section 3.4.

2.4. CONCLUSION :

This chapter was mainly devoted to the derivation of the equations used in the design calculation. In Section 2.1 the waveguide dimension was chosen mainly based on two considerations. Firstly, the flatness in propagation factor with respect to frequency and secondly the mode suppression.

In Section 2.2, we considered the problem of cavity design and TE_{01n} mode was found suitable mainly for two reasons : a) it has a high Q per volume, b) the extraneous associated TM_{11n} mode is easy to suppress. In this section we showed that in order to have minimum self interference the mode required will be TE_{011}. The dimensions of the cavity were chosen on the basis of the cross-interference.

In Section 2.3 we have chosen magnetic coupling by means of aperture since it is most suitable for TE_{011} mode. The aperture was located at the centre of the curved wall so that unwanted TM_{111} mode is suppressed. An expression for selecting aperture radius has been developed in this section.

The informations available in this Chapter are used in the next Chapter for design calculations.

DESIGN AND CALCULATION OF THE WAVEGUIDE, CAVITY AND APERTURE

3.1. DESIGN OF THE WAVEGUIDE CARRYING DOMINANT TE_{11} MODE

Based on the discussion given in Section 2.1.2, we chose for operating frequency a value 20% higher than that required for cut-off.

$$\frac{f_o}{f_c} = \frac{\lambda_c}{\lambda_o} = 120\% = 1.20 \qquad \ldots (3.1.1)$$

$$\therefore \quad (\lambda_c)_{TE_{11}} = 1.20\ \lambda_o = 4.5 \text{ cm} \qquad \ldots (3.1.2)$$

From expression (2.1.10) we find

$$(\lambda_c)_{TE_{11}} = 3.41a$$

$$\therefore \text{ diameter} = 2a = 2 \times 4.5/3.41 \text{ cm} = 2.639 \text{ cm}$$
$$= 1.039'' \qquad \ldots (3.1.3)$$

With a mode separation from next higher order mode

$$= (2.611 \times 1.2)/3.41 = 0.92 \qquad \ldots (3.1.4)$$

or 8% below.

We find the nearest value available in the standard American EIA type waveguides is

WC 109

Diameter = 1.094" ± 0.001"

Roundness Tolerance = 0.0011

Nominal outside diameter = 1.194"

Wall Thickness = 0.05 ± 0.002"

For this e waveguide we find that

$$a = 1.389 \text{ cm}$$

$$f_a/f_c = 1.263$$

Hence, the operating frequency is 26.3% higher, whereas the mode operation from next higher order TM_{01} Mode

$$= \frac{2.611 \times 1.263}{3.4} = 0.96 \qquad \ldots (3.1.5)$$

about 4%

$$\text{Since } \alpha = \frac{R_s}{a_o^x Z_o} \frac{1}{\sqrt{1 - (f_c/f_o)^2}} \left[\left(\frac{f_c}{f_o}\right)^2 + 0.420 \right] \qquad \ldots (3.1.6)$$

From error analysis we find that the variation in will be given by

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta a}{a} \left\{ 1 + \frac{(0.432)^2}{a^2\left[1 - \left(\frac{f_c}{f_o}\right)^2\right]} + \frac{2 \times (0.432)^2}{a^2\left[\left(\frac{f_c}{f_o}\right)^2 + 0.420\right]} \right\} \qquad \ldots (3.1.7)$$

Since $f_o/f_c = 1.263$ and $a = 1.094''$ $\Delta a = 0.001''$

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta a}{a}(1.710) = 0.157\% \qquad \ldots (3.1.8)$$

Hence for the tolerance specified the waveguide works like a non-dispersive medium.

3.2. DESIGN OF THE CAVITY RESONATING IN TE_{011} MODE

From the mode chart, we chose a diameter to length ratio, so that any crossing mode or interfering mode is avoided in the operating rectangular area. By careful examination of Mode Chart we find that a suitable value for $(\frac{2a_c}{d})^2 \simeq 2.00.$

$$\text{or} \quad (2a_c/d)^2 = 2.10$$

$$(2a_c/d) = 1.45 \qquad \ldots \quad (3.2.1)$$

The corresponding value for $(2a_c f)^2$ is found by using equation (2.2.9) and found

$$(2a_c f)^2 = 18.13 \times 10^8 \, (MC)^2 (cm)^2$$

$$\text{and} \quad 2a_c\, f = 4.258 \times 10^{10} \text{ Hz} - \text{cm} \qquad \ldots \quad (3.2.2)$$

Since the resonance frequency is fixed to $f = 8$ GHz

$$2a_c = 5.322 \text{ cm} = 2.095'' \qquad \ldots \quad (3.2.3)$$

For simplicity of machining we chose a value for the diameter equal to

$$\text{diameter of the cavity} = 2a_c = 2.1'' = 5.334 \text{ cm} \quad \ldots \quad (3.2.4)$$

Hence the corresponding value for

$$(2a_c f)^2 = 18.20 \times 10^8 \, (MC)^2 (cm)^2$$

$$(\frac{2a_c}{d})^2 = 2.133$$

$$2a_c/d = 1.461$$

$$\therefore d = 2a_c/1.461 = 1.44'' \qquad \ldots \quad (3.2.5)$$

For low loss on the cavity wall and hence high Q, we chose for the thickness of the wall a value ten to fifteen times the skin depth.

The skin depth is given by the formula,

$$\delta_s = (2/\omega\mu\sigma)^{1/2} \quad \ldots \quad (3.2.6)$$

Brass with 90% copper has a conductivity equal to 2.41×10^7 mho/m has a skin depth of

$$\delta_s = 1.228 \times 10^{-6} \text{ m} \quad \ldots \quad (3.2.7)$$

However, for Brass with 70% copper has a conductivity equal to 1.45×10^7 mho/m and the skin depth is

$$1.478 \times 10^{-6} \text{ m.} \quad \ldots \quad (3.2.8)$$

For this quality of brass the adequate thickness of wall from electrical considerations will be about 0.0029 cm. However, from the consideration of mechanical strength we see that this thickness will be highly inadequate, for purposes where the system might be subjected to mechanical vibration.

Hence, we chose a thickness of 0.1" for this wall, which gives the cavity a nominal outside diameter of 2.3".

However, the thickness of the wall near the aperture is made smaller since a thickwall at the aperture can cause severe attenuation and might modify the coupling, since we chose the radius of aperture from the derivation based on the assumption that the wall thickness near the aperture is small.

A suitable value for cavity wall thickness near this aperture will be equal to the thickness of the guide wall, that is equal to 0.05".

3.3. CALCULATION OF THE UNLOADED Q OF THE CAVITY :

Referring back to the section 2.2.2 equation (2.2.4) we find that the Mode Shape Factor of a cavity resonating in TE_{011} mode is given by

$$Q\ \delta_s/\lambda_o = \frac{\left[(3.832)^2 + \frac{\pi^2}{4}\left(\frac{2a_c}{d}\right)^2\right]^{3/2}}{2\pi\left[(3.832)^2 + \frac{\pi^2}{4}\left(\frac{2a_c}{d}\right)^3\right]} \quad \ldots \quad (3.3.1)$$

Since $(2a_c/d)^2 = 2.133$, the Mode Shape Factor of this cavity will be,

$$Q\ \delta_s/\lambda_o = 0.6341 \quad \ldots \quad (3.3.2)$$

Now λ_o for an operating frequency of 8 GHz will be given by

$$\lambda_o = 0.0375 \text{ m.} \quad \ldots \quad (3.3.3)$$

The skin depth δ_s, will be given by the relation

$$\delta_s = (2/\omega\mu\sigma)^{1/2} \quad \ldots \quad (3.3.4)$$

and is dependent on the material of the wall or the coating on the wall.

Silver (100% pure) will have a conductivity of 6.10×10^7 mho/m and a skin depth of

$$\delta_s = 0.721 \times 10^{-6} \text{ m.}$$

However, as calculated in section 3.2, Brass (90% copper) has a skin depth of 1.228×10^{-6}m and Brass (with a 70% copper) has a skin depth of 1.478×10^{-6}m.

Hence, the unloaded Q's of the cavity for different materials of the cavity wall are given by the equation,

$$Q = \frac{0.6341 \ \lambda_g}{\delta_s} = \frac{23.7787 \times 10^{-3}}{\delta_s} \qquad \ldots \quad (3.3.5)$$

For silver coated brass, the value for Q is

Q (silver coated brass) = 32980.16.

Similarly,

Q(Brass - 90% copper) = 19363.76

Q(Brass - 70% copper) = 16088.43

These values for Q are the unloaded Q of the cavity. When the cavity is loaded by a waveguide, the Q will reduce further. Moreover, these Q calculations assume that the inside surface finish of the cavity is very good. But because of roughness of the cavity wall, i.e., irregularity, there will be some loss reducing the Q by about 10% to 20%.

3.4. DESIGN OF THE APERTURE COUPLING THE WAVEGUIDE TO THE CAVITY :

Referring back to section 2.3.2 and figure 4 giving the equivalent diagram, we find that the condition of critical coupling is that the impedance referred to the primary side at resonance be matched to the characteristic impedance of the waveguide.

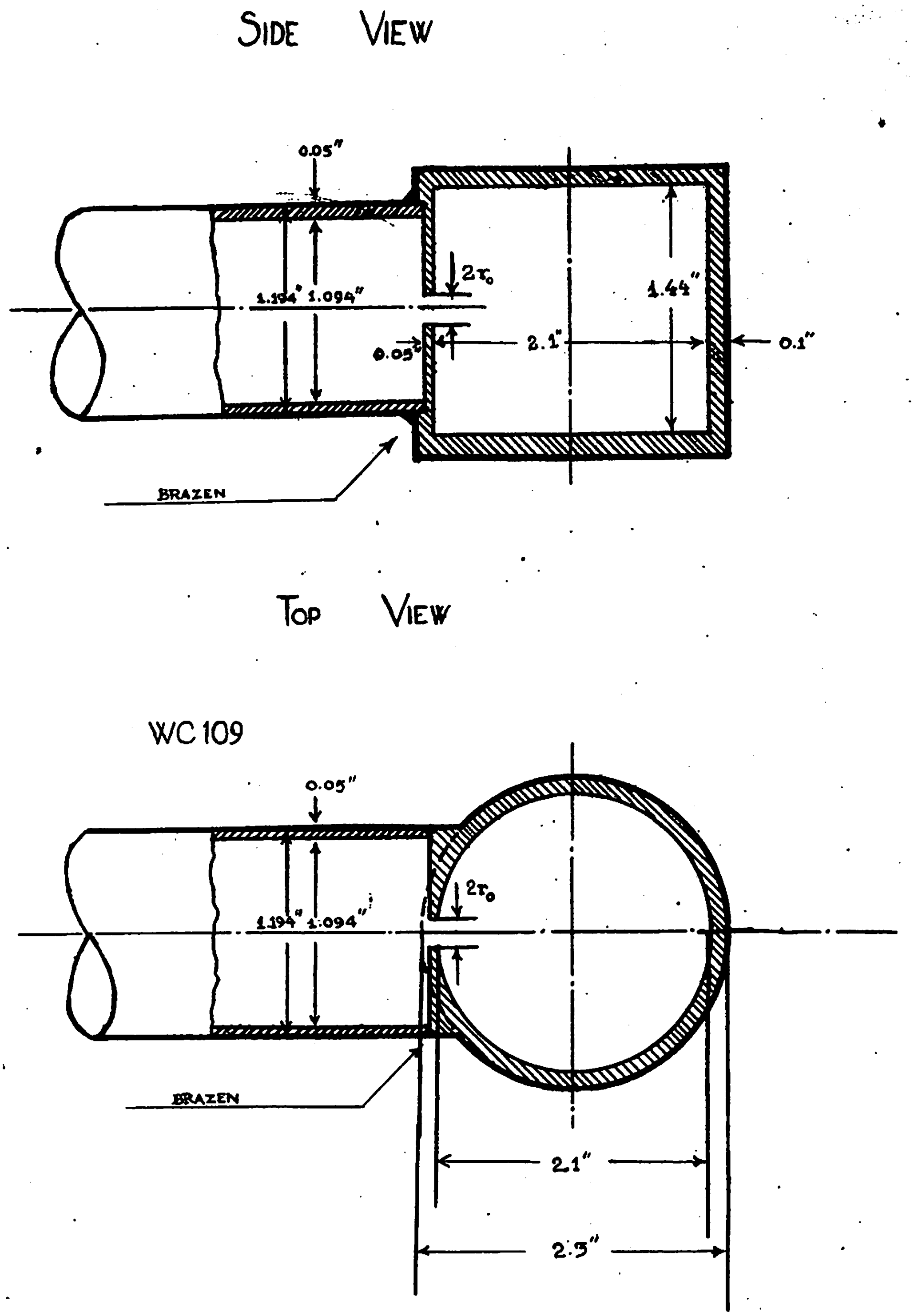


FIGURE NO. 7	SCALE : FULL
FABRICATION DRAWING (CAVITY RESONATES IN TE_{011} MODE)	
DRAWN BY : B. MISRA.	

Diameter = 1.094" ± 0.001"

Roundness Tolerance = 0.0011

Nominal outside Diameter = 1.194"

Wall Thickness = 0.05 ± 0.002"

b) CAVITY

Diameter = 2.1"

Length = 1.44"

Nominal outside Diameter = 2.3"

Wall Thickness = 0.1"

Wall Thickness near the aperture = 0.05"

c) APERTURE

Radius of the aperture for Silver
Coated Brass Cavity = 0.0753"

Radius of the aperture for Brass(90% copper) = 0.0861"

Radius of the aperture for Brass(70% copper) = 0.0901"

d) The cavity will resonate at TE_{011} mode, whereas associated TM_{11} mode will be suppressed by means of the coupling used.

e) THE UNLOADED Q OF THE CAVITY

Q (unloaded) for silver coated Brass Cavity = 32980.16

Q (unloaded)for Brass (90% copper) = 19363.76

Q (unloaded) for Brass (70% copper) = 16088.43

f) THE LOADED Q OF THE CAVITY

Since we have chosen critical coupling loaded Q will be equal to half of the unloaded Q.

Q (loaded) for Silver coated Brass cavity = 16490.1

Q (loaded) for Brass (90% copper) = 9681.9

Q (loaded) for Brass (70% copper) = 8044.2

AN ALTERNATIVE SCHEME USING TE_{111} MODE :

The cavity resonating in TE_{011} mode and coupled to waveguide transmitting TE_{11} mode has been considered in details in previous two chapters. However, because of problem of space it might be necessary that the waveguide be coupled to the cavity by means of a centred circular aperture, where the aperture is at the centre of the transverse walls of both waveguide as well as the cavity.

In this chapter a very brief theory with calculations of cavity dimensions and aperture dimensions will be made. Though it has advantages like the ease with which this can be fabricated, the simplicity in design, and immunity of coupling coefficient to variation of direction of polarization, it will be shown to be inferior to the previous scheme in many ways, such as, low Q, inadequate degenerate mode suppression and thus strengthening our previous choice.

4.1. THE FIELD EQUATIONS AND DESIGN OF THE CAVITY :

The lowest resonant mode in a cylindrical cavity is TE_{111} mode corresponding to the dominant TE_{11} mode in the circular guides. The field equations are

$$H_z = J_1\left(\frac{p'_{11}\rho}{a}\right)\cos\phi \left\{-2j \sin\beta_{11}z\right\}$$

$$H_\rho = \frac{-j\,\beta_{11}a}{p'_{11}} J'_1\left(\frac{p'_{11}\rho}{a}\right)\cos\phi \left\{2\cos\beta_{11}z\right\} \qquad \ldots (4.1.1$$

$$H_\phi = \frac{j\,\beta_{11}a^2}{(p'_{11})^2} J_1\left(\frac{p'_{11}\rho}{a}\right)\sin\phi \left\{2\cos\beta_{11}z\right\}$$

$$\left.\begin{aligned} E_\rho &= \frac{jk_o z_o a^2}{(p'_{11})^2 \rho} J_1 \left(\frac{p'_{11}\rho}{a}\right) \sin\phi \quad \left\{-2j \sin \beta_{11} z\right\} \\ E_\phi &= \frac{jk_o z_o a}{p'_{11}} J'_1 \left(\frac{p'_{11}\rho}{a}\right) \cos\phi \quad \left\{-2j \sin \beta_{11} z\right\} \\ E_z &= 0 \end{aligned}\right\} \quad \ldots \quad (4.1.1)$$

where we find from the boundary condition that

$$\beta_{11} = \pi/d. \qquad \ldots \quad (4.1.2)$$

The resonant frequency of this mode is given by the equation

$$\frac{2\pi f_{111}}{c} = \left[\left(\frac{\pi}{d}\right)^2 + \left(\frac{p'_{11}}{a}\right)^2\right]^{1/2} \qquad \ldots \quad (4.1.3)$$

The mode shape factor for TE_{111} mode is given by

$$Q\frac{\delta_s}{\lambda_o} = \frac{\left[1 - \left(\frac{1}{p'_{11}}\right)^2\right] \left[(p'_{11})^2 + \left(\frac{\pi a}{d}\right)^2\right]^{3/2}}{2\pi\left[(p'_{11})^2 + \frac{2a}{d}\left(\frac{\pi a}{d}\right)^2 + \left(1 - \frac{2a}{d}\right)\left(\frac{\pi a}{p'_{11} d}\right)^2\right]} \qquad \ldots \quad (4.1.4)$$

CALCULATION :

For simplicity of fabrication we assume that the cavity has an inner diameter equal to that of the waveguide. For a frequency of 8 GHz, we can calculate the length of the cavity for a diameter equal to 2a = 1.094" = 2.779 cm.

Using equation 4.1.3, we calculate a value of d equal to

d = 3.068 cm = 1.21"

The factor of square of diameter to length ratio for these dimensions is found to be 0.8205. By examining the Mode Chart we find that this might give cross interference with TM_{010} mode. For the cavity dimensions as designed, the TM_{010} mode can be sustained at a resonance frequency given by

$$\frac{2\pi f_{11}}{c} = \left(\frac{p_{01}}{a}\right)$$

Putting the value for the cavity dimension we obtain a resonance frequency of

8.269 GHz

which can give rise to mode degeneracy, since it is sufficiently close to operating frequency. However, the aperture at the centre of transverse wall can excite TE_{111} mode while TM_{010} is suppressed.

The mode shape factor for the cavity is calculated by using 4.1.4 and found equal to,

$$Q\ \delta_s/\lambda_o = 0.2678.$$

And the skin depth for different materials are used to give quality factor of cavities of different materials.

Unloaded Q for cavity of Silver (100%) coated Brass = 13948

Unloaded Q for cavity of Brass (90% copper) = 8757

Unloaded Q for cavity of Brass (70% copper) = 6792.5.

It is clearly seen that the Q of the cavity is much lower relative to that of a cavity resonating in TE_{011} mode.

4.2. DESIGN OF THE APERTURE :

The coupling arrangement here is chosen on the following three basis :

a) The coupling is chosen such that it excites the mode wanted in the cavity while suppressing the unwanted mode(s).

b) The coefficient of coupling is chosen for critical coupling.

The first condition is satisfied by choosing the aperture at the centre of the transverse wall of the cavity, since under this arrangement, the TE_{111} mode with maximum magnetic field at the centre is excited whereas the TM_{010} with magnetic field at the centre equal to zero is suppressed.

The calculation of the aperture radius with an aim to obtain critical coupling is found out as below.

A small circular aperture in the transverse wall behaves as a shunt inductive susceptance with a normalised value given by

$$B = \frac{-0.955\, a^2 \lambda_g}{4\, \alpha_m} \qquad \ldots \quad (4.2.1)$$

where a = waveguide radius

λ_g = guide wavelength

$\alpha_m = 4/3\ l^3$

l = radius of the aperture.

Using the equivalent - transmission-line-circuit the input impedance offered by the coupled cavity near the aperture is given by parallel impedance of $j\bar{X}_L$ and $j \tan \beta d$ and is

$$\bar{Z}_{in} = \frac{-\bar{X}_L \tan \beta d}{j\bar{X}_L + j \tan \beta d} \qquad \ldots (4.2.2)$$

Now applying a Taylor series expansion to $(\omega - \omega_1)\, \bar{Z}_{in}(\omega)$ we obtain

$$\bar{Z}_{in}(\omega) = \lim_{\omega \to \omega_1} \frac{(\omega - \omega_1)\, \bar{Z}_{in}(\omega)}{\omega - \omega_1} + \frac{d}{d\omega}(\omega - \omega_1)\, \bar{Z}_{in}\Big|_{\omega} + \cdots \qquad \ldots (4.2.3)$$

By expanding the denominator of (4.2.2) in taylor series in β about β_1, and using the valid assumptions such as

$\bar{X}_{L1} \ll 1$ and $\beta_1 d \approx \pi$, we obtain

$$\bar{Z}_{in} = -j \frac{\bar{X}_{L1}^2}{\beta_1' d(\omega - \omega_1)} \qquad \ldots (4.2.4)$$

where $\beta_1' = \frac{d\beta}{d\omega}\Big|_{\omega = \omega_1}$ for cylindrical waveguide is

$\omega_1 / \beta_1 c^2$.

For a high Q cavity the losses may be accounted for by replacing the resonant frequency ω_1 by a complex resonant frequency $\omega_1(1 + j/2Q)$. For the lossy case we have

$$\bar{Z}_{in} = -j \frac{\bar{X}_{L1}^2}{\beta_1' d(\omega - \omega_1 - j\,\omega_1/2Q)} \quad \ldots (4.2.5)$$

At resonance, we obtain a pure resistive impedance $\bar{R}_{in}$ given by

$$\bar{R}_{in} = \bar{Z}_{in} = \frac{2\bar{X}_{L1}^2 Q}{\omega_1 \beta_1' d} \quad \ldots (4.2.6)$$

If we want the cavity to be matched to the waveguide at resonance, we must choose aperture reactance $\bar{X}_{L1}$ so that $\bar{R}_{in} = 1$; that is

$$\bar{X}_{L1} = \left(\frac{\omega_1 \beta_1' d}{2Q}\right)^{1/2} \quad \ldots (4.2.7)$$

This equation gives us a method of calculating the aperture radius for critical coupling.

CALCULATION :

For this cavity, we find

$$\beta_1 = \sqrt{k_o^2 - \left(\frac{p_{11}'}{a}\right)^2} = 1.02361 \text{ rad/sec} \quad \ldots (4.2.8)$$

and $\beta_1' = \omega_1 / \beta_1 c^2 = 5.453 \times 10^{-11}$ sec/cm

The guide wavelength is

$$\lambda_g = 6.135 \text{ cm.}$$

Using expression (4.2.7) and (4.2.1) we obtain

$$0.4718 \; l^3 = \left(\frac{4.2002}{Q}\right)^{1/2}$$

$$\therefore l = \left(\frac{18.8692}{Q}\right)^{1/6} \quad \ldots (4.2.9)$$

For critical coupling the radius of aperture for different cavities of different materials are

Radius of aperture for silver coated

Brass cavity = 0.333 cm = 0.131"

Radius of aperture for Brass

(90% copper) cavity = 0.359 cm = 0.141"

Radius of aperture for Brass

(70% copper) cavity = 0.376 cm = 0.148".

4.3. CONCLUSION :

The dimensions chosen for the cavity and aperture are as follows :

1. Cavity :

   Diameter of the cavity = 1.094"

   Nominal outside diameter = 1.194"

   Wall Thickness = 0.05."

   Length of the cavity = 1.21."

2. Aperture :

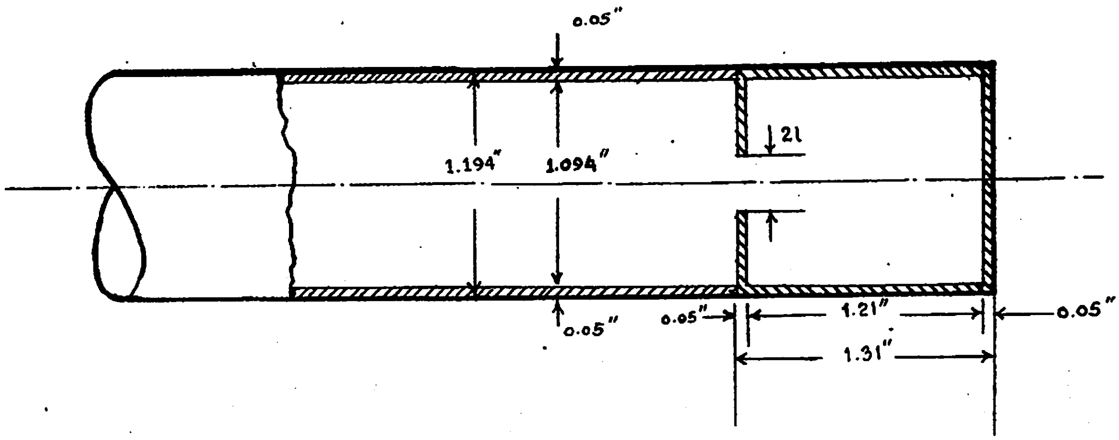
   Radius of the aperture for

   (i) Silver coates brass cavity = 0.131"

   (ii) Brass (90% copper) = 0.141"

   (iii) Brass (70% copper) = 0.148".

3. The mode that will resonate is TE_{111} mode. While it can have crossing mode interference with nearby TM_{010} mode, this is suppressed by proper choice of coupling.



CROSS SECTION

FIGURE No: 8.	SCALE : FULL
FABRICATION DRAWING (CAVITY RESONATES IN TE_{111} MODE)	
DRAWN BY : B. MISRA.	

4. The loaded Q for

(i) Silver coated Brass cavity = 13948

(ii) Brass (90% copper) cavity = 8767

(iii) Brass (70% copper) cavity = 6792.5

While comparing this system of waveguide coupled to cavity resonating in TE_{111} mode, we find the advantages of this over the other in two respects -

a) It is simple to fabricate.

b) Since this has axial symmetry, even if the polarization of the wave in the waveguide changes because of ellipticity, the coupling remains critical, whereas in the previous scheme if the axis of the cavity is not parallel to direction of polarisation, the coupling is not perfect.

However, this scheme has two major disadvantages, such as,

a) The quality factor of this cavity is not high.

b) Suppression of unwanted modes is difficult.

DISCUSSION AND CONCLUSION :

5.1. CARE TO BE TAKEN WHILE FABRICATING :

The fabrication of cavity to be coupled to the waveguide forms an important part of the main work and is to be done carefully, because, otherwise it will be difficult to achieve the high Q, obtained from the design.

In the design the effect of roughness of the inner side of the cavity wall was not taken into account, because of its complicated nature. These irregularities on the wall increase the amount of power-loss and hence reduce the Q. Though, there does not exist any simple quantitative relation showing dependence of Q on the cavity wall roughness, it is found from experiments that when the irregularities are much smaller than one tenth of the wavelength of the existing mode, the effect of these in reducing Q is very small. Hence, while choosing a cutting tool to surface the cavity wall, the effort should be made to reduce the roughness below this value.

The geometry of structure is very important in realizing the potential Q of the cavity. The theoretical computations are based on a perfect right circular cylinder and the design is based on these, in practice it is seldom achieved. Distortion occurs in various forms: e.g., a) the cylinder instead of being round may be elliptical; b) the ends may not be perpendicular to the axis of the cylinder or c) not parallel to each other; and as considered before d) surface irregularities may be present causing distortion within the cavity. While these factors are not under the control of the designer, an effort should be made to minimize these effects by requiring adherence to close dimensional tolerance.

In most of the designs, the requirement on the parallelism of the end plates is more stringent than can be commercially produced. An adjustable mechanism for levelling is a practical solution in case of TE_{011} mode, since it has $H_\phi = 0$ and hence there are no axial currents, and the end plates of the cavity can be free to move. Tilt adjustment in the order of 0.001 inch at the edge of the plate is required in the 8 GHz band.

In case of the design in which the inner surface of cavity is plated by silver coating, adequate control of the conductivity of the interior surfaces of the cavity is necessary to achieve a uniform manufactured product.. This requires attention not only to thickness and uniformity of the plating but also purity of plating bath and avoidance of foreign matters during buffing processes. Since silver (100%) has a skin depth at this frequency (8 GHz) equal to 0.721×10^{-6} m, we chose a thickness for silver coat equal to about 40 times the skin depth or 1.0 mil.

In case of cavity resonating in TE_{011} mode and coupled to a waveguide transmitting in TE_{11} mode there is another problem to be taken into account. The guide is generally excited at a particular polarisation and this might change as the electromagnetic wave propagates along the guide, as a result of ellipticity. However, it is necessary that near the aperture the magnetic field on guide side should be parallel to the magnetic field on the cavity side for maximum coupling. Hence the cavity has to be adjusted with a very stringent tolerance, to achieve this.

Moreover, the circular iris is to be made on the cavity and waveguide transverse wall with a very low tolerance, since with slight change, the coefficient of coupling can change very much. The surface near the aperture is to be machined so that the thickness of the wall is not high since this can give high attenuation and modify coupling coefficient. In addition, care should be taken to see that surface near the aperture does not have any discontinuity.

The waveguide and cavity are to be connected together and soldered or brazed so that there is no leakage at the connecting walls. The scheme is presented in fig.7.

5.2. IMPROVEMENT IN MODE SUPPRESSION AND CAVITY Q :

In case of cavity at TE_{011} mode coupled to waveguide in TE_{11} mode, by centred circular aperture, the location of the aperture was chosen at the centre of the cylindrical wall of the cavity so that it can suppress the unwanted accompanying TM_{111} mode. However, if the mode suppression is not complete by this arrangement, another method can be adopted to suppress TM_{111} mode in the cavity.

The end plate can be modified to include a gap at its periphery. The gap perturbs the resonant frequencies of the two modes by different amounts so that they become separated. Secondly, the peripheral gap cuts through the surface currents at points of high density for TM mode and minimum density for TE_{011} mode and hence forms a mode suppressor.

However, the cavity with a peripheral gap may give rise to spurious resonance in the region behind the reflecting surfaces if the responses are not damped. The addition of a lossy material such as bakelite or carbon loaded neoprene in the back cavity is a successful suppression method.

From the field configuration of TE_{011} mode we see that the electrical field lines are parallel to the cavity wall whereas magnetic field lines as perpendicular. As a result of this anomalous field configuration, it is found that a dielectric coating may be put on the wall of the cavity if necessary to reduce the loss and improve the Q.

5.3. EFFECT OF TEMPERATURE ON FREQUENCY OF RESONANCE :

The cavity frequency of resonance depends upon the temperature.

a) For a cavity constructed out of single metal the % change in resonant wavelength is equal to percentage change in linear dimension and hence coefficient of material of which the cavity is built.

b) If the cavity is not sealed, there will be a change in resonant frequency because of varying dielectric constant of air with changing temperature.

Since the coefficient of thermal expansion of brass is 19×10^{-6} per °C and the rate of change of humidity in air is low, for reasonable amount of temperature change, resonant frequency can be assumed constant.

5.4. CONCLUSION :

In this design project, based on the types of application in which a cavity is used a strategy for solution was assumed. The cavity mode was selected for high Q whereas coupling arrangement was selected for critical coupling.

The waveguide was designed for an almost uniform attenuation factor within the operating frequency range and for adequate mode separation.

The cavity was designed assuming TE_{011} mode. The Q of the cavity was calculated assuming a perfect cylinder with a very smooth surface.

The effect of roughness of wall, ellipticity of cylinder non parallel shorting walls in reducing the cavity Q was not considered because of complexity involved.

Based on assumptions such as - a) a very small iris on a thin wall with no surface discontinuity, and b) the cavity Q is the Q assumed for perfect cylinder, the dimension of the aperture was calculated, for critical coupling.

In none of these design the effect of temperature and humidity was considered, since the range of temperature variation was assumed to be small.

An alternative scheme was presented with a cavity resonating in TE_{111} mode and the pros and cons of this scheme were weighed against and the previous scheme was found better in many respects.

The mode charts, field distribution, fabrication drawings, computer results were given.

APPENDIX :

TABLE A.1

PHYSICAL CONSTANTS REFERRED TO IN THE DESIGN

Permittivity of vacuum = ϵ_o = 8.854×10^{-12} Farad/m

Permeability of vacuum = μ_o = 4×10^{-7} henry/m

Impedance of free space = Z_o = 376.7 ohms.

Velocity of light = C = 2.998×10^{8} m/sec.

TABLE A.2

CONDUCTIVITY OF MATERIALS USED

Material	Conductivity ($\times 10^{7}$ mho/m)
Silver (100%)	6.10
Copper (100%)	5.80
Silver (7½% copper)	5.20
Aluminium (100%)	3.43
Brass (90% copper)	2.41
Magnesium (100%)	2.06
Brass (70% copper)	1.45

TABLE A.3

COEFFICIENT OF THERMAL EXPANSION AT 20°C

Brass	19×10^{-6}
Copper	19×10^{-6}

TABLE A.4

PROPERTIES OF BESSEL FUNCTIONS

$$J_n(kr) = \sum_{m=0}^{\infty} \frac{(-1)^m \, (kr/2)^{n+2m}}{m! \, (n+m)!}$$

TABLE A.4 (contd.)

$$J_{n-1}(kr) - J_{n+1}(kr) = 2J_n'(kr)$$

$$J_0'(kr) = -J_1(kr)$$

$$J_1'(kr) = J_0(kr) - \frac{1}{kr} J_1(kr)$$

TABLE A.5

ROOTS OF BESSEL FUNCTIONS :

n	p_{n1}	p_{n2}	p_{n3}	p_{n1}'	p_{n2}'	p_{n3}'
0	2.405	5.520	8.654	3.832	7.016	10.174
1	3.832	7.016	10.174	1.841	5.331	8.536
2	5.135	8.417	11.620	3.054	6.706	9.970

TABLE A.6

LIST OF SYMBOLS USED

H = Magnetic Field

E = Electric Field

ω = Angular Frequency

γ = Propagation constant

α = Attenuation Factor

β = Phase Shift Factor

μ = Permeability of free space

ϵ = Permittivity of free space

c = Speed of light

a = radius of the waveguide

a_c = radius of the cavity

d = length of the cavity

TABLE A.6 (contd.)

r_o, l	=	radius of aperture
J_n	=	Bessel function of 1st kind and nth order
p_{nm}	=	mth root of J_n
p'_{nm}	=	mth root of J'_n
Z	=	Wave Impedance
k	=	Wave number
λ	=	Wave length
f_c	=	cut off frequency
λ_c	=	cut-off wave length
δ_s	=	skin depth
σ	=	conductivity of a material
Q	=	quality factor
N	=	Total number of resonant frequencies less than f
V	=	volume of cavity
M	=	$Q\delta_s/\lambda_o$ = Mode shape Factor
e^a	=	Mode Function
V_o	=	Dominant Mode Voltage
n	=	turns ratio of equivalent ideal transformer
Z_o	=	characteristic impedance

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