WORKHOLDING

Analysis and Planning

Bud Mishra

Robotics and Manufacturing Research Laboratory
Courant Institute, New York University

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Abstract

With an increasing interest in manufacturing towards computer-assisted production in small batch sizes, researchers have begun to focus on the design, analysis and planning algorithms for workholding (also called fixturing*), calibration, tool-path generation. In this paper, we focus on the problem of workholding and review how various recently developed qualitative and quantitative approaches of grasping theory can be applied to this problem* mutatis mutandis. *However, the problems in workholding differ from robot-hand grasping problems in two fundamental ways, namely: (1) The accuracy and ambient force (resulting from fluctuating cutting loads) requirements are rather stringent (2) The available fixtures are geometrically more specialized as compared to robot-hands. In the concluding section, we reflect on how these differences lead to further interesting and challenging questions.*

1. Introduction

In standard manufacturing systems, workholding devices are used to immobilize specific parts, while the final shape of the part is imparted by successive phases of cutting processes. In addition to immobilizing the structure, the workholding devices (usually, clamps, jigs and fixtures) must be so placed that they do not interfere with the tool paths or the calibration process. Additionally, the forces imparted by the jigs and fixtures must not exceed certain thresholds in order not to deform or destroy the stock. But, simultaneously they must guarantee that the parts being machined are immobilized to the extent that the results of the calibration and machining is within the tolerable accuracies.

Note, however, that unlike the traditional robotics structures objects in the manufacturing systems are required to withstand extremely high fluctuating cutting loads. For instance, it is quite common to require an accuracy of less than 0.025 mm and cutting loads as high as 20 KN. The problems relating to workholding has received a lot of attention in the recent years (see for instance, [1, 3, 4, 5, 6, 8, 11].) However, most of the research in the area focuses on expert-system approaches, and thus, relies on existing heuristics, many of which are yet to be justified from an analytical viewpoint. For instance, Boyes[2] offers the following rule for workholding with toe clamps: *The number of contacts between the part and the locating surface(s) must be greater than or eual to three in one plane and greater than or equal to two in another plane perpendicular to the first.* However, it is unclear if such a rule is applicable to any arbitrary stock subject to other kinematics constraints. Some knowledge-based approaches have succeeded in solving the part setup problems in limited domain; for instance, the sequence of setups shown in Figure 1 are generated by an expert system. Yet, it is not immediate how such techniques can be generalized to more complicated domains.

Figure 1: A sequence of “setups” used in cleaning up a rectangular stock. In each setup, the stock has been fixtured using toe clamps.
Courtsey: Paul Englert & Paul Wright. A Planning System for Part Setup and Workholding, *Adv. Manuf. Eng.*, Vol 1, pp. 249–263, October 1989.

In this paper, we focus on the problem of workholding and review how various recently developed qualitative and quantitative approaches of grasping theory can be applied to this problem.

For the sake of concreteness, we shall concentrate on the “small-batch milling,” where a clamping is usually chosen by a human operator for workholding. These devices usually comprise toe clamps and toggle clamps, which fit into the toe slots on the bed of the machine tool and then grab onto the part at convenient projections. In order to keep the deformation minimal these clamps are assumed to have a rather small area of contact, which we may conveniently approximate to a nominal point-of-contact. Additionally, as the objects to be machined can be assumed to be obtained from some solid-modeler, we may assume that such objects are full-bodied and compact.

2. Immobility

Let the part to be clamped be denoted by B and assumed to have a piece-wise smooth boundary ∂B. We let ∂B^* denote the set of points $\mathbf{p} \in \partial B$ such that the direction $\mathbf{n}(\mathbf{p})$ normal to ∂B at $\mathbf{p}$ is well-defined; by convention, we pick $\mathbf{n}(\mathbf{p})$ to be the unit normal pointing into the interior of B.

With each nominal point of contact (corresponding to a clamp), we can associate a twist system; they describe the degrees of freedom of the body local to that contact point. Equivalently, we can describe the clamping structure, by means of a system of n wrenches, say:

$$\{\mathbf{w}_1, \ldots, \mathbf{w}_k, \mathbf{w}_{k+1}, \ldots, \mathbf{w}_n\},$$

the first k of which are bisense and the remaining last $n-k$ of the wrenches are unisense. One interesting special case occurs if we assume no friction at the contacts. In this case the force $\mathbf{f}$ at each contact point $\mathbf{p}$ must be in the direction of the surface normal $\mathbf{n}(\mathbf{p})$; the wrench system associated with each point is:

$$\Gamma(\mathbf{p}) = \{[\mathbf{n}(\mathbf{p}), \mathbf{p} \times \mathbf{n}(\mathbf{p})]\},$$

and all the wrenches are unisense, as we must have magnitude of $\mathbf{f} = \mathbf{f} \cdot \mathbf{n}(\mathbf{p}) \geq 0$.

Now, a system with a set of contacts is free to move by an external twist if and only if the virtual coefficients of the twist and the contact wrenches is nonnegative, since otherwise the virtual work done by the wrench would be negative. This occurs when the twist is *reciprocal* to the bisense wrenches and *reciprocal* or *repelling* to the unisense wrenches. A set of twists (associated with the contacts) is said to *immobilize* (or constitutes a *form closure*) if and only if any arbitrary twist is resisted by the set of contacts.

Thus if $\mathbf{d}$ is an arbitrary twist then it must be *non-reciprocal* to some $\mathbf{w}_i$ $(i = 1, \ldots, k)$ or must be *contrary* to some $\mathbf{w}_j$ $(j = k+1, \ldots, n)$. Put another way, this is equivalent to saying that, for any arbitrary vector $\mathbf{d}'$, we have

$$\begin{aligned} \mathbf{w}_i \cdot \mathbf{d}' &= 0, \quad \text{for all } 1 \leq i \leq k \qquad \text{implies that} \\ \mathbf{w}_j \cdot \mathbf{d}' = \pi\mathbf{w}_j \cdot \mathbf{d}' &< 0, \quad \text{for some } k+1 \leq i \leq n, \end{aligned}$$

which, in turn is equivalent to the condition that

$$\mathbf{0} \in \text{int conv } (\pi\mathbf{w}_{k+1}, \ldots, \pi\mathbf{w}_n), \tag{1}$$

where π is the linear projection function onto $L^\perp$ whose kernel is L = the linear space lin $(\mathbf{w}_1, \ldots, \mathbf{w}_k)$. [By $L^\perp$, we mean the orthogonal complement of L.]

By this process, one has converted a problem in mechanics into a purely geometric problem, now amenable to many interesting techniques in convexity theory and computational and combinatorial geometry. It is further known that the geometric condition given in equation (1) is also a necessary and sufficient condition for *force/torque closure*—a concept widely studied in grasping theory (see [9] and [10]). Recall that a system of wrenches

$\mathbf{w}_1, \ldots, \mathbf{w}_n$ (as before) constitutes a *force/torque closure* if and only if any arbitrary external wrench can be generated by varying the magnitudes of the wrenches (subject to the constraints imposed by the senses of the wrenches). A necessary and sufficient condition for a closure grasp is that the (module) sum of the linear space spanned by the vectors $\mathbf{w}_1$, $\ldots$, $\mathbf{w}_k$ and the positive space spanned by the vectors $\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n$ is the entire $\mathbb{R}^{\mathbb{D}}$:

$$\text{lin } (\mathbf{w}_1, \ldots, \mathbf{w}_k) + \text{pos } (\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n) = \mathbb{R}^{\mathbb{D}}, \tag{2}$$

where D is the dimension of the wrench space. The equation (2) is equivalent to the condition

$$\text{lin } (\mathbf{w}_1, \ldots, \mathbf{w}_k) + \text{pos } (\pi\mathbf{w}_{k+1}, \ldots, \pi\mathbf{w}_n) = \mathbb{R}^{\mathbb{D}},$$

and thus to the condition in equation (1).

These observations can be very fruitfully exploited in devising algorithms to synthesize clamping structures that immobilizes a part.

3. Case of No Friction

For the simple case where the contacts are non-frictional, based on the work of Mishra et. al. [9], one can obtain several interesting results. Although, the condition of "no-friction" seems physically unattractive, this stringent condition allows for the situations where the available friction is unreliably small or unknown.

Based on the classical results of Reuleaux and Somoff, we know that in order to immobilize an object we need at least $D+1$ clamps: that is, a two dimensional object requires at least *four* clamps and a three dimensional object requires at least *seven*.

One can modify the results of Mishra et. al. [9] to obtain the most general bounds on the number of clamps. They have also provided an algorithm that finds *at least one* immobilization for a polygonal/polyhedral object and their algorithm runs in time linear in the number of edges/faces of the object. In the following paragraphs, we briefly describe the techniques of Mishra et. al.

Recall that, in the non-frictional case, the force on the object at a contact point must be along the surface-normal, directed inward. In this situation, we have a *wrench map*, Γ, mapping ∂B^* into the D-dimensional *wrench space* $\mathbb{R}^{\mathbb{D}}$ as follows:

$$\Gamma \colon \partial B^* \to \mathbb{R}^{\mathbb{D}} \ : \mathbf{p} \mapsto [\mathbf{n}(\mathbf{p}), \mathbf{p} \times \mathbf{n}(\mathbf{p})].$$

Essentially, Γ maps $\mathbf{p}$ to the point $\Gamma(\mathbf{p})$ in the wrench space that represents the effects of applying a unit force at $\mathbf{p}$ in the direction $\mathbf{n}(\mathbf{p})$.

Now, given an arbitrary compact rigid object B whose piece-wise smooth boundary ∂B is not a circle (in 2-D) or a surface of revolution (in 3-D), it can be shown that

Theorem 1:
B can be immobilized by a clamping structure of at most $2D$ clamps.

The assumption that ∂B is not a circle or a surface of revolution is essential, since, in the absence of friction, no matter how many clamps are employed the object can always be rotated about its axis of symmetry.

By virtue of the discussion of the previous section, we only need to show that

$$\Big(\exists\ \mathbf{p}_1, \ldots, \mathbf{p}_k \in \partial B^*,\ k \leq 2D\Big)$$
$$\Big[\mathbf{0} \in \text{int conv}\ \Big(\Gamma(\mathbf{p}_1), \ldots, \Gamma(\mathbf{p}_k)\Big)\Big].$$

The proof is in two steps:
(1) We claim that

$$\mathbf{0} \in \text{int conv}\ \Gamma(\partial B^*).$$

Using Gauss' divergence theorem, we see that

$$\left[\int_{\partial B^*} \mathbf{n}(\mathbf{p})\, dS,\ \int_{\partial B^*} (\mathbf{p} \times \mathbf{n}(\mathbf{p}))\, dS\right] = \mathbf{0},$$

i.e.

$$\mathbf{0} \in \text{conv}\ \Gamma(\partial B^*).$$

Thus, it remains to show that the origin is indeed an interior point of the convex hull; assume to the contrary. Then it can be shown that there is a nonzero vector $\mathbf{g} = [F, \tau] \in \mathbb{R}^{\mathbb{D}}$ orthogonal to the linear subspace spanned by $\Gamma(\partial B^*)$. Thus, for each function $c(\mathbf{p})$ we have

$$\int_{\partial B^*} c(\mathbf{p})\ [(F + \tau \times \mathbf{p}) \cdot \mathbf{n}(\mathbf{p})]\ dS = 0.$$

In particular, substitute the function $c(\mathbf{p}) = (F + \tau \times \mathbf{p}) \cdot \mathbf{n}(\mathbf{p})$ into the last equation, to deduce that $(F + \tau \times \mathbf{p}) \cdot \mathbf{n}(\mathbf{p})$ must be identically 0 over ∂B^*. But, this is possible only in the case when the boundary of B is a circle or a surface of revolution.
(2) The rest follows by an application of the following theorem from combinatorial geometry:

> **Steinitz's Theorem.** [9]
>
> *If* $X \subseteq \mathbb{R}$ *and* $\mathbf{p} \in \text{int conv}\ X$*, then* $\mathbf{p} \in \text{int conv}\ Y$ *for some* $Y \subseteq X$ *with* $|Y| \leq 2d$.

Our main result then follows from Steinitz's Theorem, once we identify X with $\Gamma(\partial B^*)$, and Y with the set $\{\Gamma(\mathbf{p}_1), \ldots, \Gamma(\mathbf{p}_k)\}$, with $k \leq 2 \times D$.

Additionally, using a simple perturbation argument, one can improve the number of clamps to $D+1$: in that case the number of clamps is optimal, since it matches the bounds given by Reuleaux and Somoff.

4. $2\frac{1}{2}$-D Rectilinear Objects

The theory discussed in earlier sections assumes that the clamps can be applied at any point on the boundary of the part (subject only to few minor exceptions). In reality, however, almost all workholding devices are required to obey certain severe geometric and kinematic constraints. For instance, the positions of toe clamps (and hence the contact points) that can be selected are constrained by the positions of the toe slots on the bed of the machine tool.

Let us consider a simple arrangement of the toe slots: *The toe slots are arranged along the vertices of a unit grid on the bed of the machine tool.* The toe clamps fit into the toe slots and grab on to the part with a small projection.

In many cases, the part B to be machined can be assumed to be two-and-a-half dimensional, i.e. the object is cylindrical with a polygonal cross-section and of relatively small height. A toe clamp fixed to a toe slot, TS, will be able to make a contact on a vertical face within a unit distance of TS and at a point closest to TS. Note that while a toe slot TS can potentially contribute more than one contact point, in any clamping structure, at most one such contact point can be used.

Thus, it is clear that the geometric constraints imposed by the toe slots and the kinematic constraints imposed by the toe clamps, make it infeasible to apply the techniques of the last two sections directly. However, it is still possible to obtain several interesting results for these situations, only using techniques from combinatorial geometry.

The following theorems are for the case when the part is assumed to be *rectilinear*; that is, its cross-section is a rectilinear polygon with each edge perpendicular or parallel to the others. We state these theorems without proof; the techniques employed to prove these theorems are similar to the ones in Mishra et. al. [9].

> **Theorem 2:**
> *Let B be a $2\frac{1}{2}$-D rectilinear part whose cross-sectional polygon has edges all of length 4 units or more.*
>
> *Then B can be clamped by at most 6 toe clamps that fit into a unit grid of toe slots.*

The proof of the theorem is straightforward. First note that as a result of the assumption on the length and by *Pigeonhole Principle*, it can be shown that one can put two clamps per edge such that the contact points straddle the midpoint of the edge. Thus, if the part has N faces, we have a clamping structure with $2N - 4$ toe clamps. Furthermore, it can be shown that the $2N - 4$ toe clamps immobilize the part. Now, by using the *reduction procedure* of Mishra et. al. [9], we can reduce the number of clamps to 6. The algorithm to synthesize such a clamping structure can be implemented to run in time linear in N.

Cleanup: One important operation in manufacturing is to take a part that is approximately the desired finished product (within the tolerance) and to clean up each face. The cleanup operation takes a sequence of setups: each setup comprising of a clamping process, followed by a cutting process. Usually one tries to minimize the number of setups, while trying to keep the operations within each setup fairly simple (few clamps and few toolpaths).

Theorem 3:
Let B be a $2\frac{1}{2}$-D rectilinear part whose cross-sectional polygon has edges all of length 6 units or more.

Then B can be cleaned up in 4 setups—each setup uses at most 6 toe clamps that fit into a unit grid of toe slots.

Part Creation: Lastly, we consider the operation of creating a part from a rectangular stock. As before, this operation requires a sequence of setups, and one tries to minimize the number of setups.

Theorem 4:
Let B be a $2\frac{1}{2}$-D rectilinear part whose cross-sectional polygon has edges all of length 10 units or more.

Then B can be constructed from a rectangular stock in $O(N)$ setups—each setup uses at most 12 toe clamps that fit into a unit grid of toe slots. Here, N is the number of faces.

The algorithms for constructing each setup in both the processes can be implemented in time linear in the number of faces of the parts.

It is not clear if the theorems in this section produce clamping structures, with optimal number of clamps, with optimal number of setups, etc. More research needs to be done to answer these questions. Also, it is not obvious how our techniques can be generalized to non-rectilinear objects.

5. Concluding Remarks

In a manner similar to our work on "grasp-metrics" [7], one can also introduce certain quantitative criteria for designing "good" workholding and clamping structures. However, some of the geometric techniques, inherent to [7], can also be further sharpened to design the most efficient workholding that avoids the toolpath but can withstand the cutting loads associated with the particular toolpath.

There is also a great need for the problem of characterizing the stability of the parts in various workholding systems. Besides static stability (as emphasized here), one needs to study the effect buckling instabilities and vibrations.

The questions related to the facts that the objects of interest are determined by the underlying "constructive solid geometry" of the modeler and that each phase of cutting requires design and planning of the reclamping, etc. pose further challenges.

Lastly, manufacturing science seems to be based purely on a large class of sample case studies for practicing machinists. As the manufacturing processes begin to be computerized and automated, one needs to study the analytical and algorithmic aspects of the subject. One hopes that this paper stimulates such efforts.

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