FORMATION OF ROUTH'S ARRAY IN THE PRESENCE OF A ROW OF ZEROES

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While forming Routh's Array used for the determination of the stability of a system, it is sometimes advantageous to introduce an infinitesimally small shift of the imaginary axis. This can avoid a situation resulting in a row of zeroes, since it introduces asymmetry.

This method is also useful in using Routh-Hurwitz criterion to find out the root distribution correctly in the left-hand plane (LHP), on the imaginary axis and in the right-hand plane (RHP).

An algorithm is developed for forming the array taking the first-order approximation only.

Indexing Terms : Routh's Array, System Stability

IN THE special case under consideration, when all the elements in one row of the Routh Tabulation are zeroes, it indicates that one or more of the following conditions may exist: (*a*) Pairs of roots with opposite signs (*b*) Pairs of imaginary roots, opposite signs (*c*) Pairs of complex conjugate roots forming symmetry about the origin of the *s*-plane.

Hence this situation can be avoided, if we introduce asymmetry by shifting the imaginary axis slightly to the left or right.

The same method is also suggested (Rao and Rao 1975) to avoid incorrect results due to random movement of the roots on the imaginary axis into LHP or RHP, when an ε is substituted for a zero first element. Let the new plane after the shift of the imaginary axis be called S'-Plane. In this new plane, the polynomial will be obtained by replacing S by $S'+\varepsilon$, where ε is a real quantity, positive or negative according as the axis is shifted to right or left respectively. The Routh's array be formed with the parameter ε appearing in all terms.

New, if we keep ε positive but make it approach zero from the right in the limit (*i.e.*, in the limit $\varepsilon \rightarrow 0+$) the number of changes in sign will give the number of poles in RHP. Similarly, if $\varepsilon \rightarrow 0-$ in the limit, we will get the number of poles in the RHP as well as on imaginary axis.

The method is shown for a polynomial of n with first order approximation in ε. Since ε is an infinitesimally small real quantity, in the analysis higher-order terms of ε are neglected.

After the shift of axis, the following replacements are used;

$$S = S' + \varepsilon$$

$$S^2 = (S' + \varepsilon)^2 = S'^2 + 2\varepsilon S'$$

$$\vdots$$

$$S^n = (S' + \varepsilon)^n = S'^n + n\varepsilon S'^{n-1}$$

The first two rows are formed as usual, but with ε appearing in each term.

Once the first two rows are formed, the subsequent rows can be easily found. We consider two possible cases.

Case 1

For the Routh's array after the shift of the imaginary axis, let the two rows of the array be given by

$$\begin{matrix} b_0 + c_0\varepsilon & b_2 + c_2\varepsilon \\ b_1 + c_1\varepsilon & b_3 + c_3\varepsilon \end{matrix}$$

Now the first term of the next row will be given according to the Routh's Rule, by ;

$$d_1 - \frac{1}{b_1}\left[c_1 d_1 - \{(b_1 c_2 - b_0 c_3) + (c_1 b_2 - b_3 c_0)\}\right]\varepsilon$$

where, $d_1 = (b_1 b_2 - b_0 b_3)/b_1$

where, the higher-order terms are neglected.

A careful observation shows that the first term d_1 is as it should be when ε is zero, whereas the co-efficient of ε can be easily formed by using a rule similar to the one used in forming d_1.

Case 2

But, if b_1 and b_3 are zeroes, the above array has to be rearranged and the subsequent terms are found as follows :

$$\begin{matrix} b_0 + c_0\varepsilon & b_2 + c_2\varepsilon \\ c_1\varepsilon & c_3\varepsilon \end{matrix}$$

Then the first term of the next row is

$$\frac{c_1 b_2 - c_3 b_0}{c_1} + \frac{c_1 c_2 - c_3 c_0}{c_1}\varepsilon$$

Again the formation of this term shows a close resemblance to the Routh's Rule.

Now the number of poles in the RHP can be found from the number of changes in sign of the co-efficients of the first column as ε is made to tend to zero.

Example

Let the polynomial of denominator be

$$S^6 + S^5 - 2S^4 - 3S^3 - 7S^2 - 4S - 4 = 0$$

The Routh's array will be found by taking the first power of ε;

$\varepsilon \rightarrow 0-$	$\varepsilon \rightarrow 0+$					
+	+	S^6	1	$-2+5\varepsilon$	$-7-9\varepsilon$	$-4-4\varepsilon$
+	+	S^5	$1+6\varepsilon$	$-3-8\varepsilon$	$-4-14\varepsilon$	
+	+	S^4	$1-5\varepsilon$	$-3-19\varepsilon$	$-4-4\varepsilon$	
−	+	S^3	44ε	34ε		
−	−	S^2	$-3{\cdot}77-15.13\varepsilon,$	$-4-4\varepsilon$		
+	−	S^1	-12.66ε			
−	−	S^0	$-4-4\varepsilon$			

Noting the changes in sign for lim $\varepsilon \rightarrow 0+$ & lim $\varepsilon \rightarrow 0-$, we note that there is a pole in RHP and three poles in RHP & imaginary axis. Hence this closed-loop transfer function has a single pole in RHP & imaginary pole-pair on the imaginary axis.

REFERENCES

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