April 7 2015

LECTURE # 8

GAME THEORY

Study of strategic interactions ...
- Choices
- Rational Decisions

Strategic Choices.

- Static Games
- Dynamic Games { Signalling, Bargaining
- Evolutionary Games.

Issues:
- Privacy (Information Asymmetry)
- Trust (Correlation of Encounter)
  - Anonimity
  - Liquidity
- Signaling
- Coordination
  - Bargaining
  - Pricing
  - Auctioning.

Advertisement

User \ Advertiser	No Ad	Show Ad
$\neg$ Click	(0, 0)	(0, -1)
Click $\wedge$ $\neg$ Buy	(-1, 0)	(-1, -1)
Click $\wedge$ Buy	(1, 2)	(1, 1)

Note that in the absence of any information on the user, the advertiser is better off by <u>not</u> showing any ad.

$\Rightarrow$ Solution: User Information $\begin{cases} \text{Cookie} \\ \text{Key-word} \\ \text{3rd party data.} \end{cases}$

· You need two games (Signaling Games).

User $\Leftrightarrow$ Publisher $\Longleftrightarrow$ Advertisers.

Key-word. (Signals) — User-data (Cookie).

SEARCH (Browser) — AUCTION (Ad-Exchange)

SSP (Ad Networks) $\longrightarrow$ RTB Real Time Bidding $\longleftarrow$ DSP (Agencies)

GAME THEORY.

KEY ASSUMPTIONS (often violated)

1) Rationality. (or Bounded Rationality)
2) CKR: Common Knowledge of Rationality.

An Individual → knows his options/strategies
→ knows his utilities/payoffs.
→ knows he is rational (i.e. utility optimizing)
→ knows similar information about the other player(s)
→ knows that others ara rational and that others know that he is rational ... etc.

He strategically selects an option that optimizes his payoff. (while ~~at~~ anticipating how others will behave.)

Note: ◦ Payoff need not be just monetary
◦ Rationality should be treated as an idealization:
- Learning
- Statistical Inference/Data Science/Computation
- Evolutionary Stable Strategies.

Strategic Form Games.

Defn: A strategic form game is a triplet.

$$\langle I, (S_i)_{i \in I}, (u_i)_{i \in I} \rangle$$

such that:

a) Index Set: $I \equiv$ Finite set of players.
$\equiv \{1, 2, 3, \ldots, \ell\}$

b) Strategy Set: S_i, $i \in I \equiv$ Set of available actions for player $i \in I$.

c) Strategy Profile:
$$S = \prod_i S_i$$

d) Utility Function:
$$u_i : S \rightarrow \mathbb{R}$$

$\equiv$ The payoff function of player $i \in I$.

Notation $\quad S \equiv \prod_i S_i \;;\; s_i \in S_i$

$$S_{-i} = \prod_{j \neq i} S_j \equiv \text{Strategy Profile for all players except } i \in I.$$

$$\therefore \; S = S_i \times S_{-i} \;;\; s_{-i} \in S_{-i}$$

$s_{-i} = \langle s_j \rangle_{j \neq i} \equiv$ Vector of all actions for all players excluding player $i \in I$.

$\langle s_i, s_{-i} \rangle \equiv$ Strategy Profile.

Example: BoS (Battle of the Sexes)

$\equiv$ Two Player Game.

$I = \{F, M\}$ Two players of opposite sex
M = Male; F = Female.

$S_F = S_M = \{O, F\}$ O: Opera
F = Football.

$$S = S_F \times S_M = \{\langle O,O\rangle, \langle O,F\rangle, \langle F,O\rangle, \langle F,F\rangle\}$$

F = Row Player M = Column Player

F \ M	O	F
O	3,2	0,0
F	0,0	2,3

$u_F : S \to \mathbb{R}$ $u_M : S \to \mathbb{R}$

$$u_F(\langle O,O\rangle) = u_M(\langle F,F\rangle) = 3$$
$$u_F(\langle F,F\rangle) = u_M(\langle O,O\rangle) = 2$$

Otherwise
0.

PLAY OF THE GAME

Each player chooses a strategy s_i

$$\Downarrow$$

Strategy Profile = $\langle s_1, s_2, \cdots, s_\ell \rangle \equiv s$

$$\Downarrow$$

Utility = $u_i(s)$

If $s^* = \langle s_1^*, s_2^*, \ldots, s_\ell^* \rangle$ = "BEST"

then

$$\forall_{i \in I} \ \forall_{s_i \in S_i}$$

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$$

$\boxed{\text{Best Response}}$

① STABILITY: No player can profitably deviate given the strategy of the other players.

② FIXED POINT UNDER CKR: Each player chooses a strategy (s_i^*) expecting all other players to choose "rationally."

NASH EQUILIBRIUM

(PURE STRATEGY N.E.)

A pure strategy Nash Equilibrium of a strategic game:

$$\langle I, (S_i)_{i\in I}, (u_i)_{i\in I} \rangle$$

is a strategy profile $s^* \in S$ such that

$$\forall_{i\in I} \forall_{s_i \in S_i} \quad u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$$

Example BoS:

- GREEDY STRATEGY $\not\Rightarrow$ NE.

  s_F = opera, s_M = football $\Rightarrow$ payoff = (0,0)

  Either s_F should deviate to football:

  payoff $0 \to 2$.

  or s_M should deviate to opera

  payoff $0 \to 2$.

- ULTRA ALTRUISTIC STRATEGY $\not\Rightarrow$ NE.

  s_F = football, s_M = Opera $\Rightarrow$ payoff = (0,0)

  Either s_F should deviate to opera

  payoff $0 \to 3$

  or. s_M should deviate to football

  payoff $0 \to 3$

There exist

TWO NASH EQUILIBRIA (Pure Strategy)

$\langle O, O \rangle$ or $\langle F, F \rangle$

$\langle$opera, opero$\rangle$ $\langle$football, foot ball$\rangle$

F enjoys both the opera & M's company
M gains utility by being in F's company.

Example. Beautiful Mind.

JN \ JNF	Blonde	Brunnette
Blonde	(0,0)	(2,1)
Brunnette	(1,2)	(½,½)

In the movie JNF selects brunnette
$\Rightarrow$ Not Rational!
Where is the Nash Equilibrium?

Example (Extended) Prisoner's Dilemma (P.D.)

P1 \ P2	Confess	Silence	suicide
Confess	(-2, -2)	(0, -3)	(-2, -10)
Silence	(-3, 0)	(-1, -1)	(0, -10)
Suicide	(-10, -2)	(-10, 0)	(-10, -10)

1) Suicide is dominated
$\Rightarrow$ Eliminate.

2) silence is dominated
$\Rightarrow$ Eliminate.

$\Downarrow$

⟨Confess, Confess⟩
$\equiv$ P.S.N.E.

Rock-Paper-Scissors.

$\nexists$ P.S.N.E.

P_1 \ P_2	R	P	S
R	(0,0)	(-1, 1)	(1, -1)
P	(1, -1)	(0, 0)	(-1, 1)
S	(-1, 1)	(1, -1)	(0, 0)

ZERO-SUM GAME.

$\Downarrow$

MIXED STRATEGY.

NOTE BOOK

MIXED-STRATEGY

Σ_i = Prob. Measure over Pure Strategies S_i

$\sigma_i = (p_{i_1}, p_{i_2}, \cdots, p_{i_K})$

$p_{i_j} = Pr[\, S_{i_j} \in S_i$ is played$\,]$

$p_{i_j} \geq 0 \quad \sum p_{i_j} = 1.$

$\Sigma = \prod_i \Sigma_i \triangleq$ Mixed Strategy Profile.

$\sigma \in \Sigma$

$$u_i(\sigma) = \int_S u_i(s)\, d\sigma(s) = \sum p_{i_j} u_i(s_{i_j}, \sigma_{-i})$$

MIXED STRATEGY NASH EQUILIBRIUM:

$\sigma^* \in \Sigma \equiv$ M.S.N.E.

iff

$$\forall_i \; \forall_{\sigma_i \in \Sigma_i} \quad u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$$

NASH's THM ($\because$ Kakutani Fixed Point Thm.)

Every finite game has a Mixed Strategy Nash Equilibrium.