

$G = (V, E) = \left. \begin{array}{l} \text{Undirected} \\ \text{Directed} \end{array} \right\} \text{graph.}$

Connected Component:

A connected component of a graph is defined as a maximal subgraph in which path exists from every node to every other.

◊ A path in a graph is closed if its start and end vertices coincide:

$$v_0, v_1, \dots, v_n \equiv v_0$$

$$\forall i (v_i, v_{i+1}) \in E \quad 0 \leq i < n$$

$$\forall i, j \quad v_i \neq v_j \quad 0 \leq i < j \leq n$$

◊ A cycle is defined as a closed path in which $n \geq 3$.

Strongly Connected Component

A strongly connected component of a graph is defined as a maximal subgraph in which cycle exists connecting every node to every other.

TREE: A tree is a connected graph that contains no cycle.

ADJACENCY MATRIX.

Every graph $G = (V, E)$ ^(Undir) with $|V| = n$ has associated with it a symmetric adjacency matrix $A \in \{0, 1\}^{n \times n}$

Binary $n \times n$ matrix A in which

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ \& } v_j \text{ are adjacent} \\ & (v_i, v_j) \in E \\ 0 & \text{otherwise.} \end{cases}$$

Since in an undirected graph

$$(v_i, v_j) \equiv (v_j, v_i) \quad a_{ij} = a_{ji}$$

$$A^T = A. \quad \leftarrow \text{A real-valued symmetric matrix.}$$

$$d_{ii} = d(v_i) = |\{v_j \mid (v_i, v_j) \in E\}| = \text{Degree.}$$

$$D = \begin{bmatrix} d_{11} & & 0 \\ & d_{22} & \\ 0 & & \ddots \\ & & & d_{nn} \end{bmatrix} = \text{Diagonal Matrix.}$$

$$\text{Trace } D = \sum_{v_i} d(v_i) = 2m.$$

Boundary Matrix $B \in \{-1, 0, +1\}^{m \times n}$

Columns are indexed by the vertices of G .
Rows are indexed by the edges of G

$$B(e, v) = \begin{cases} +1 & \text{if } v \text{ is the head of } e \\ -1 & \text{if } v \text{ is the tail of } e \\ 0 & \text{otherwise} \end{cases}$$

$$B^T B = L = D - A.$$

Choose edge directions arbitrarily if $G = \text{undir.}$

$$L = D - A.$$

Random Surfing on G .

$$P(u, v) = \begin{cases} \frac{1}{d_u} & \text{if } (u, v) \in E \\ 0 & \text{otherwise} \end{cases}$$

$$P = D^{-1} A.$$

$$L = D(I - P) = D\Delta; \Delta \equiv I - P$$

$\Delta = I - P \equiv$ (Discrete) Laplace operator.

Your position within the social network assigns a social-value.

⇒ Rank
(e.g. page rank).

$$f(x)$$

f: V → R a scalar rank function.

Dirichlet Sum of G

$$\sum_{u,v \in V} \sum_{(u,v) \in E} (f(u) - f(v))^2$$

You'd like to choose ranks so that the sum is minimized:

Avoid the trivial rank: $f(x) = 1 \forall x$.

⇒ Focusing on the relative values.

$$\Delta f(x) = \frac{1}{d_x} \sum_{(y,x) \in E} (f(x) - f(y))$$



$$(I - P) f.$$

RANDOM SURFER MODEL.

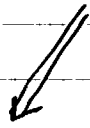
Imagine a web surfer bouncing along randomly following the graph (hyperlink graph of the web.)

- ① When the surfer arrives at a node he chooses at random hyper-links (directed edge) to a new node.

② Asymptotically, the proportion of time the random surfer spends on a given node/page is a measure of

Relevance (Relative Importance)

- ③ Dangling Nodes - Sinks } Problems.
 Periodicity in the graph }



Stochastic Teleportation.