

*Social*Networks*

QUIZ #5

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Q1. [10] The University of Facebook, UFB, declares any pair of its students to be *friends*, if they overlap in time during which they attended UFB. They allow a student to enroll anytime during the year and graduate whenever he wishes, but they do not allow a student to interrupt the period during which he is enrolled. Thus, a student A can attend UFB during $[t_A, t'_A]$, and B , during $[t_B, t'_B]$. A and B are friends if $[t_A, t'_A] \cap [t_B, t'_B] \neq \emptyset$. They have had n students.

Recall that maximal complete subgraphs of a graph are called cliques. How many cliques can a UFB social network have? How large a clique can it have? How will you enumerate all its cliques?

SOLN.1 *We can represent a UFB social network as an interval graph.*

Note that if we ensure that there is no transitivity, that is A overlaps with B and B overlaps with C , but then A does not overlap with C , then there are $O(n)$ cliques (each of size 2). This can be made to hold true for any constant sized cliques. The largest clique can be of size n : everyone enrolls and graduates at the same time. A simple way to enumerate the cliques will be to scan the intervals from left to right and use a very simple form of dynamic programming. When a new student arrives, he either starts a new clique or adds to the most recent clique; when the most recent student leaves, he simply exits a clique.

Q2. [10] Someone presents you a graph $G = (V, E)$ and tells you that it represents a UFB social network: that is, the n students are its vertices $|V| = n$ and the friend relations are its edges – $(u, v) \in E$ iff u and v are friends. How will you check this claim without any additional data (without breaching the students' privacy)?

SOLN.2 *To determine whether a given graph $G = (V, E)$ is an interval graph, your algorithm needs $O(|V| + |E|)$ time by seeking an ordering of the (maximal) cliques of G that is consecutive with respect to vertex inclusion.*