Social*Networks

QUIZ #2

B. Mishra

04 February 2014

Q1. [10] In a country where everyone wants a boy, each family continues having babies till they have a boy. After some time, what is the proportion of boys to girls in the country? (We are assuming that probability of having a boy or a girl is the same and is independent of previous child – you may want to think about what happens if this assumption is violated.)

SOLN.1 *The easiest way to see that there will be about same number of boys and girls, is to use an* **invariant principle**. *At any "round," assume that there are roughly equal number of boys and girls; then in the next "round" only a subset of couples (who have had only girls in the previous rounds) will attempt to have children and those couples who succeed will contribute roughly equal number of boys and girls to the population. Thus the invariant will be satisfied by the inductive hypothesis, assuming that it was true at round 0, when of course boys and girls were equal in number, as their number was equal to zero.*

The following is a stupider way of getting the same answer: Assume there are C number of couples so there would be C boys (expected). The expected number of girls can be calculated by the following method.

Expected Number of Girls per Couple

$$\begin{aligned} &= 0(\textit{Probability of } 0 \textit{ girls}) + 1(\textit{Probability of } 1 \textit{ girl}) + 2(\textit{Probability of } 2 \textit{ girls}) + \cdots \\ &= 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{8} \cdots \\ &\approx 1 \end{aligned}$$

Since the expected number of girls is C, *the proportion of boys to girls is* 1 : 1.

Q2. [5] What makes this problem relevant to this class?

SOLN.2 *Proof by induction will be important to us. We will also be looking for invariants in a network to understand its behavior: e.g., number of friend-requests sent will be same as number of friend-requests received. Think of all other such invariants that come to mind.*