Solution of Homework 2
Social Networks

Exercise 4.4 Let A be a square n × n matrix whose rows are orthonormal. Prove that the columns of A are orthonormal.

Solution: Let A be $\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$, the rows are orthonormal so $r_i \cdot r_j = 1\ if\ i = j\ and\ 0$ otherwise, where r_i is the i-th row of A.
The means $AA^T = A^TA = I$ since A is n*n matrix.
Therefore we can get $c_i \cdot c_j = 1\ if\ i = j\ and\ 0$ otherwise, where c_i is the i-th column of A. Thus the columns are also orthonormal.

Exercise 4.6 Prove that the left singular vectors of A are the right singular vectors of A^T.

Solution: Let $A = UDV^T$ by SVD where U is composed of the left singular values of A and V is composed of the right singular values of A. Then $A^T = (UDV^T)^T = VDU^T$, so the left singular vectors of A are the right singular vectors of A^T.

Exercise 4.31
1. Consider the pairwise distance matrix for twenty US cities given below. Use the algorithm of Exercise 2 to place the cities on a map of the US.
2. Suppose you had airline distances for 50 cities around the world. Could you use these distances to construct a world map?

Solution:
1. In order to put the cities on a map, we would compute the rank 2 approximation of the matrix of distance. This gives us a projection of the rows of the original matrix onto a two –dimensional subspace spanned by the first 2 singular vectors of the original matrix.
2. Yes, map M to a three – dimensional subspace. Do a rank 3 approximation of the distance matrix to get an approximate map of the world in three dimensions.