GAME THEORY.

Study of Strategic Interactions { Biological Systems, Economic Systems, Internet.

2 Ingredients { Choices, Rational Decisions → Utility Optimizing.

- Each agent solves a computational problem whose parameters are determined by the solutions of the other agents.
- To make the problem well-defined we will make every agent "rational"
  Weaker Versions: Common Knowledge of Rationality. (CKR)
- Boundedly Rational (Satisficing → as opposed to being optimizing).
- Evolutionarily Stable Strategies [Darwinian Evolution.].
  ~No need for epistemology~ Teleology.
- Information Asymmetric Signaling Games-
  { Deception, Scam } → { Privacy, Trust, security } !

STRATEGIC FORM GAMES

It is a triplet

$$\langle I, (S_i)_{i \in I}, (u_i)_{i \in I} \rangle$$

- I: (Finite) Index — Set of players $\{1, 2, \dots, \ell\}$
- S_i, $i \in I$: Strategy set — Set of Actions available to each player
- $u_i : S \to \mathbb{R}$: Utility — Pay off function for each player $i \in I$

$S = \Pi_i S_i$ (STRATEGY PROFILE).

BoS. (Battle of the Sexes).

TWO PLAYER GAME.

M → Male
F → Female.

F \ M	Opera	Football
Opera	3,2	0,0
Football	0,0	2,3

$I = \{F, M\}$

$S_F = S_M = \{\text{Opera, Football}\}$

$S = \{\text{Opera, Football}\}^2$

$u_F : S \to \mathbb{R}$ $\quad u_M : S \to \mathbb{R}$

$u_F(\langle \text{Opera, Opera} \rangle) = u_M(\langle \text{Football, Football} \rangle) = 3$

$u_F(\langle \text{Football, Football} \rangle) = u_M(\langle \text{Opera, Opera} \rangle) = 2$

$\{\text{O. W.} \longrightarrow 0\}$

Notation :

$S = \prod_i S_i$ = Set of all action profiles.

$s_i \in S_i$ = An action available to player i.

$S_{-i} = \prod_{j \neq i} S_i$ = Strategy profiles for all players except player i.

$S = S_i \times S_{-i}$

$s_{-i} \in S_{-i}$

$\langle s_i, s_{-i} \rangle$ = A strategy profile.

"Best Strategy"

$$s^* = \langle s_1^*, s_2^*, \dots, s_i^* \rangle$$

Each player makes his best response.

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall_i \quad \forall_{s_i \in S_i}$$

↳ PURE STRATEGY NASH EQUILIBRIUM.

BoS ⇒ It has two P.S.N.E. { ⟨Opera, Opera⟩, ⟨Football, Football⟩

(Extended) Prisoner's Dilemma (P.D.)

P1 \ P2	Confess	Silence	Suicide
Confess	(-2, -2)	(0, -3)	(-2, -10)
Silence	(-3, 0)	(-1, -1)	(0, -10)
Suicide	(-10, -2)	(-10, 0)	(-10, -10)

1) Suicide is dominated
↳ Eliminate

2) Silence is dominated
↳ Eliminate

⇓

⟨Confess, Confess⟩.
= P.S.N.E.

Rock-Paper-Scissors.

$\Rightarrow$ NO P.S.N.E.

P_1 \ P_2	R	P	S
R	(0,0)	(-1,1)	(1,-1)
P	(1,-1)	(0,0)	(-1,1)
S	(-1,1)	(1,-1)	(0,0)

← Zero-Sum Game

$\Downarrow$

Mixed-Strategy.

Σ_i = Prob. measure over Pure Strategies S_i

$\sigma_i = (p_{i_1}, \cdots, p_{i_K})$

$p_{ij} = Pr[S_{ij} \in S_i \text{ is played}]$

$p_{ij} \geq 0$, $\sum p_{ij} = 1$

$\Sigma = \prod_i \Sigma_i$ = Mixed Strategy Profile.

$\sigma \in \Sigma$

$$u_i(\sigma) = \int_S u_i(s)\, d\sigma(s) = \sum p_{ij}\, u_i(s_{ij}, \sigma_{-i})$$

Mixed Strategy Nash Equilibrium:

$\sigma^* \in \Sigma$

$$\forall_i \ \forall_{\sigma_i \in \Sigma_i} \quad u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$$

NASH'S THM

Every finite game has a MIXED STRATEGY NASH EQ.

∵ Kakutani Fixed Point Theorem.

SIGNALING GAMES.

2 Players { S = Sender; R = Receiver }

Nature selects → TYPE t_i from $T = \{t_1, \dots, t_I\}$

S sends a message to T after observing type →

MESSAGE m_j from $M = \{m_1, \dots, m_J\}$

R observes message and takes an action →

ACTION a_k from $A = \{a_1, \dots, a_K\}$

$$\text{Payoffs} = \begin{cases} U_S(t_i, m_j, a_k) \\ U_R(t_i, m_j, a_k) \end{cases}$$

Continuous time / Repeated Games.

Asynchronous.

FLIP IT. : Two players : { Owner – White Hat; Pwner – Black Hat. }

At any time $t \in [0,T]$, each player may use a strategy - SWITH - to change the state of the world.

state $\in$ {White, Black}

O: S → S : White ⟼ White, Black ⟼ White Cost = $1.

P: S → S : Black ⟼ Black, White ⟼ Black Cost = $10.

Pay-off: O: $\frac{T_{white}}{T} \times \$100 - \#O\text{-switches} \times \$1$

P. $T_{Black}/T \times \$100 - \#P\text{-switches} \times \$10$

PWN IT. Information Asymmetric

O: Cannot see the state, but P can.
Strategy Space is increased:

P demands ransom: Ransomware cost $20. ($15 cost)
If the state is Black, P can carry out the threat and cause harm to O. To restore the state O will need to spend $25.
P can bluff.
O can pay or call the bluff.

State = Black. O pays P
$-\$20$ $+\$20 - \$15 = \$5$
O does not pay P
$-\$25$ $+\$0 - \$15 = -\$15$

State = White O pays P
$-\$20$ $+\$20 - \$15 = \$5$
O does not pay P
$\$0$ $+\$0 - \$15 = -\$15$

(We can ignore the payoff from state occupancy).

PWN IT with Costly Signaling:

M-Coins: O can prove that he is "good" by being able to switch the state to white.
M-coins are like BITCOINS except that they expire.

Verifiers: Randomly challenges a player to give it a coin
He can evict a player, if it cannot produce a coin.