Lecture #10

Nov 20 2014.

CANCER MEGAFUND
(Andy Lo, MIT).

Financial Instruments.

a) Allocation of Capital
(Financing Projects: Cure for Cancer).

b) Allocation of Risk
- Diversification
- Hedging.
- Securitization/Derivatives.

c) Market for Investors with different Investment Needs.
- Insurance
- Swaps (E.g. Credit Default Swaps)
- Retirement Funds (401K)
- Hedge Funds.

d) Consumption and Smoothing.
- Saving
- Borrowing.

Problems:

Information-Asymmetry (Signaling Games)
Asynchrony (Time Arbitrage)

{ Insider Inform^n
{ Market Manip^n
Toxic Assets.

Short Sales
(Naked Shorts)

SECURITIZATION

\$1000 I.O.U $\xrightarrow{90\%}$ Payment = \$1000.00

\$1000 I.O.U $\xrightarrow{10\%}$ Default = \$0.00

Expected Value of the Bond = \$900.00
= 0.9 × \$1000.00
+ 0.1 × \$0.00.

But Market may not clear as the buyer may not be willing to take a risk of default for no clear gain.

A government bond with 11% interest will yield \$999.00 with no risk

"Sure Thing Principle."

Portfolio

Two Bonds (with independent default risks)

\$1000 I.O.U $\xrightarrow{90\%}$ Payment = \$1000.00

\$1000 I.O.U $\xrightarrow{10\%}$ Default = \$0.00

\$1000 I.O.U. $\xrightarrow{90\%}$ Payment = \$1000.00

\$1000 I.O.U. $\xrightarrow{10\%}$ Default = \$0.00

↳ Legal Entity ≡ "Special Purpose Vehicle."

Two New Claims

↳ Blue Bond ≡ Senior Tranche

↳ Orange Bond = Junior Tranche.

Both Blue and Orange Bonds have same face value (= \$ 1000.00), but different priorities/ seniorities.

Rules:

1) Blue bond is going to have a priority in terms of its seniority. Blue bond must get paid before the orange bond
2) Assume that the two bonds default in statistically independent ways. If neither ~~bond~~ IOU's defaults, both claims get paid. If one of the IOU's defaults, then only the senior (Blue) claim gets paid. If both IOU's default then neither claim (neither Blue nor Orange) gets paid.

Portfolio Value	Prob.		Senior Blue	Junior Orange
\$2000	81%	⇒	\$1000	\$1000
\$1000	18%		\$1000	\$0
\$0	1%.		\$0	\$0.

Price { Senior Tranche → $0.99 \times 1000 + 0.01 \times 0 = \$990$ (Default Risk = 1%).

Junior Tranche → $0.81 \times 1000 + 0.19 \times 0 = \$810$ (Default Risk = 19%).

Expected Value of the Portfolio = \$1800.

Rating of the Bonds

→ Blue Bond → AAA rating

Orange Bond → BA rating } Below Investment Grade.

Senior Tranche can be bought by Pension fund, Money Market fund, Sovereign fund. (+ Insurance e.g. AIG)

Junior Tranche → Hedge fund.

3) What if defaults become highly correlated?
[E.g. Bull market may not have correlations, but bear market may result in strong default correlation.]

Portfolio Value	Prob	Senior / Blue	Junior / Orange
\$ 2000	90%	\$ 1000	\$ 1000
\$ 0	10%	0	0

Price/Value. $\begin{cases} \text{Senior Tranche} = 0.9 \times 1000 + 0.1 \times 0 = \$900 \\ \quad (\text{Default Risk} = 10\%) \\ \text{Junior Tranche} = 0.9 \times 1000 + 0.1 \times 0 = \$900 \\ \quad (\text{Default Risk} = 10\%). \end{cases}$

⇒ Penion Fund; } lose money → 10% of their
MoneyMarket: } investment.

Hedge Fund: make money
⇒ Financial Crisis. of 2008.

4) What if one of the I.O.U's is a lemon.

Portfolio Value	Prob	Senior / Blue	Junior / Orange.
\$ 1000	90%	\$ 1000	\$ 0
\$ 0	10%	\$ 0	0

Value $\begin{cases} \text{Senior} = 0.9 \times 1000 + 0.1 \times 0 = \$900 \\ \text{Junior} = \$0. \end{cases}$

By creating a vehicle with lemons and selling it to unsuspecting clients, you can make money. (You buy for yourself portfolios that are free of lemons).

Curing Cancer.

Typical Drug Development Program:

- $200 MM out-of-pocket costs;
- 10-year approval process.
- Probability of success in oncology is 5%
- If successful, annual profits of $2B for 10-year patent.

$\$200\text{ MM}$ with $p = 5\% \rightarrow +51.0\%$ (compound rate of interest)

$\$200\text{ MM}$ with $1-p = 95\% \rightarrow -100\%$.

$E[R] = 11.9\%$
$SD[R] = 423.5\%$.

Portfolio $\rightarrow$ 150 drug-development process (uncorrelated).

- Requires $30B capital
- Assumes programs are IID (needs to be relaxed)

$E[R] = 11.9\%$.
$SD[R] = 34.6\%$

Securitization:

$$Pr[K \text{ successes}] = \binom{150}{k}\left(\frac{1}{20}\right)^{k}\left(\frac{19}{20}\right)^{150-k}.$$

$$\left.\begin{array}{lll} \text{At least } 1 \text{ hit} & = & 99.95\% \\ 2 \text{ hits} & = & 99.59\% \\ 3 \text{ hits} & = & 98.18\% \\ 4 \text{ hits} & = & 94.52\% \\ 5 \text{ hits} & = & 87.44\% \end{array}\right\} \text{Simulating Cancer Mega Fund.}$$

An. Example.

\# Assets = N (e.g. ~~Bon~~ Cancer-Bonds $N=150$)

p = Probability of payment of \$1 (e.g. $p=0.05$)
q = Probability of default (e.g. $q=0.95$)
$= 1-p$

A fair price for the portfolio of N assets $= Np$.
Variance $= Npq$
S.D $= \sqrt{Npq}$.

\# Lemons $= n$
Lemons default surely.

$N-n$ = non-lemon assets n = lemon assets.

Fair Price $= (N-n)p = Np - np$ Lemon-cost $= np$.

So if the buyers suspect n out of N (=150) are lemons, the will be willing to pay
$\frac{N-n}{N} \times 200$ MM per drug!

Securitization

- Create M special-purpose vehicles, each of which depend on D of the underlying assets.

- Each of the M claims pays $c\frac{Np}{M}$ ($c<1$, e.g. $c=\frac{2}{3}$) if the numbers of the assets that defaulted is at most $Dq + t\sqrt{Dpq}$ ($t \approx \sqrt{\log D}$)
Otherwise, payment $= 0$.

- In the absence of any lemon, the fair value of the M special purpose vehicles is very close $cNp = \frac{2}{3}Np$

- If pooling is done randomly (each SVP depend on D random assets) then the total fair value of the M SVPs is

$$cNp - o(n).$$

- What happens if Seller (who is asymmetrically informed) knows which assets are lemons.

$$S = \text{Set of Lemons}. \quad |S| = n.$$

- Pick some $m \ll M$ of the SVPs and make sure that these m SVPs over-represent lemons. That is, each one has about $\sqrt{D}$ lemons.

  $\Leftarrow$ TOXIC SVP's

- Sell toxic SVP's to clients, buy non-toxic ones in-house.
- Detecting toxic- SVP's
  Require checking every n-sized subset of N- assets and checking them for over-represent^{n}.

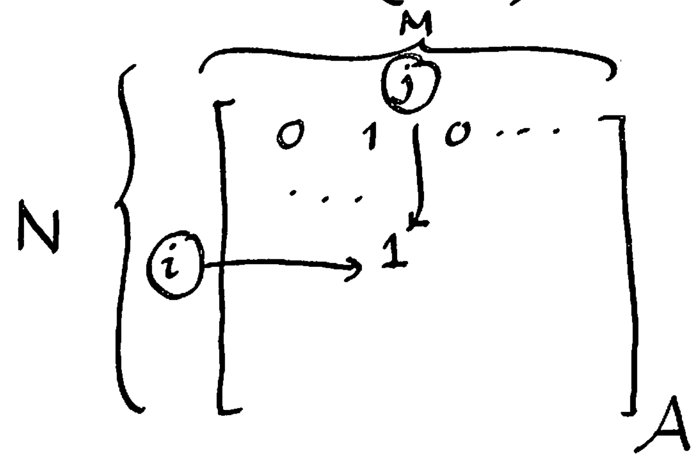
Densest Subgraph Problem.

Related to "planted clique problem"
NP- complete.

$\langle N, M, D, n, m, d \rangle \leftarrow$ Parameters.

$N = o(MD)$

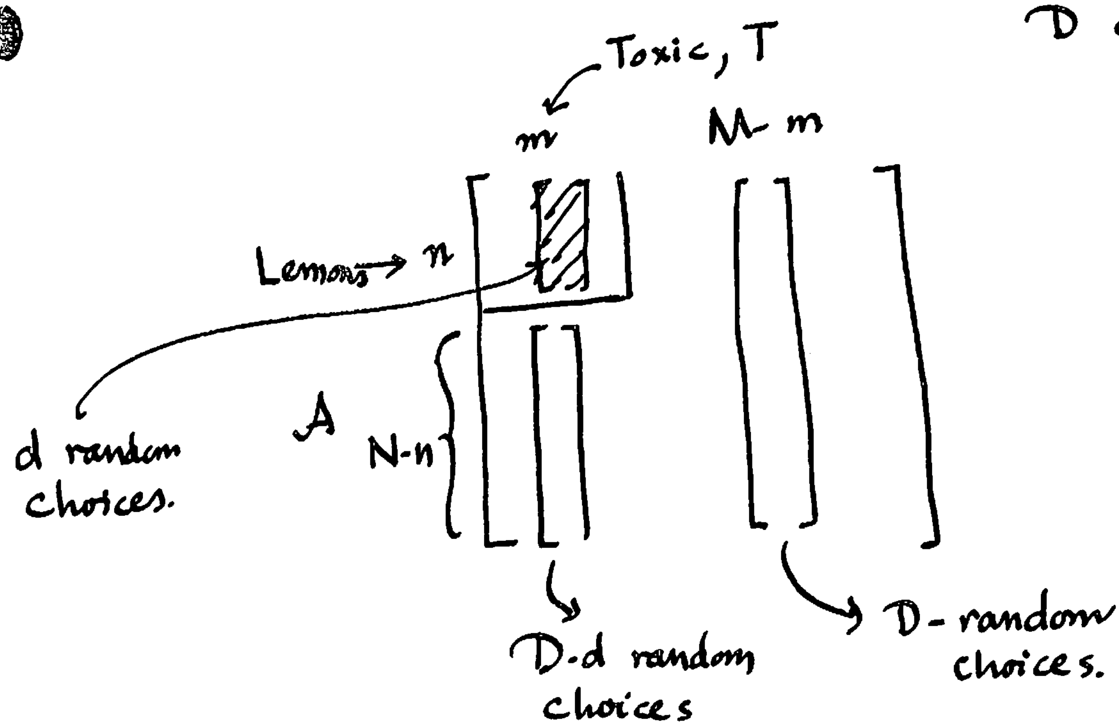
$\left(\frac{md^2}{n}\right)^2 = o\left(\frac{MD^2}{N}\right)$, $d \approx O(\sqrt{D})$



$A_{ij} = \begin{cases} 1 & \text{if } j\text{-SPV contains } i\text{-asset} \\ 0 & \text{o.w.} \end{cases}$

Column sum = D.

$\Rightarrow$ Each SPV has D assets.



Two Distributions $\longrightarrow$ Random, R

= Every column has D random 1's

$\longrightarrow$ Poisoned, P

= Every toxic column has d random lemons, and D-d random non-lem

Theorem:

There is no $\epsilon > 0$ and P-time algorithm that distinguishes between R and P with ϵ-advantage.