Lecture #10

April 16 2013

CHOICES:

1) Information to share / Website to visit...
2) Evaluating information from friends, coworkers, acquaintances, etc.
3) Accepting friendship
4) Trust
5) Sellers to buy goods from
6) Auction
7) Pricing

GAME THEORY.

Method of studying strategic decision making.

1) Static Games
2) Dynamic Games (under uncertainty)
   - → Signaling Games
   - → Bargaining Games.
3) Evolutionary Games.

KEY ASSUMPTIONS: (often violated)

1) Rationality
2) Common Knowledge of Rationality (CKR)

★ Individuals act <u>rationally</u> in the sense of choosing an option that gives them higher pay-off.

a) Payoffs need not be just monetary ~ (Social and psychological payoffs may play role.)

b) Still rational decision making paradigm is useful in providing a foundation for the theory.

- Bounded Rationality
- Evolutionary Stable Strategies.

ORDINAL INFORMATION.

a) Set of Options/Strategies:

$$S = \{s_1, s_2, \dots, s_n\}$$

b) Utility Function: (Real Valued).

$$u: S \to \mathbb{R}$$

$u(\cdot)$ represents ranking of different options:

$$u(s_{i_1}) \geq u(s_{i_2}) \geq \dots \geq u(s_{i_n})$$

c) Every choice induces a probability distribution over consequences

$$F^{s_i}(c). \quad \text{or} \quad p^{s_i}_{c_j}$$

continuous pdf. discrete prob.

d) There is a utility function, called Bernoulli Utility Function, $\mathcal{U}(c)$, which gives utility of a consequence (outcome) c.

Expected utility under uncertainty

$$u(s_i) = \sum_j p^{s_i}_{c_j} \, \mathcal{U}(c_j)$$

$$u(s_i) = \int \mathcal{U}(c) f^{s_i}(c) \, dc$$

$$= \int \mathcal{U}(c) dF^{s_i}(c).$$

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MULTIPLAYER SITUATION:

John von Neumann.
+ Oskar Morgenstern.

1) A set of "reasonable" axioms for Rational Decision Making (under uncertainty)

2) Expected Utility Theory.
Under uncertainty, every choice induces a lottery (Probability Distribution over different outcomes.)

Given two actions: s_a and s_b
→ Probability Distribution

$$\mathcal{F}^{s_a}(c) \quad \mathcal{F}^{s_b}(c)$$

Choose a over b iff

$$u(s_a) \geq u(s_b)$$

$$\int u(c)\, d\mathcal{F}^{s_a}(c) \geq \int u(c)\, d\mathcal{F}^{s_b}(c).$$

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STRATEGIC FORM GAMES:

Defn:

A strategic form game is a triplet:

$$\langle I, (S_i)_{i \in I}, (u_i)_{i \in I} \rangle$$

such that

INDEX SET I: Finite set of players indexed by $I = \{1, 2, 3, \ldots, \ell\}$

STRATEGY SET S_i: Set of available actions for player i

- Finite
- Countably Finite
- Uncountably Infinite.

$s_i \in S_i$ is an action for player i

$u_i : S \to \mathbb{R}$ is the payoff (utility) function of player i.

$S = \prod_i S_i$ is the set of all action profiles.

NASH EQUILIBRIUM.

◇ PURE STRATEGY NASH EQUILIBRIUM:

A pure strategy Nash Equilibrium of a strategic game

$$\langle I, (S_i)_{i \in I}, (u_i)_{i \in I} \rangle$$

is a strategy profile $s^* \in S$ such that

$\forall_{i \in I} \ \forall_{s_i \in S_i}$

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$$

{ Notation:

$s_{-i} = [s_j]_{j \neq i}$ { Vector of actions for all players except i

$S_{-i} = \prod_{j \neq i} S_i$ { Set of all strategy profiles for all players except i.

$S_i \times S_{-i} \equiv S$

$(s_i, s_{-i}) \in S$ is a strategy profile or Outcome

NASH EQUILIBRIUM.

1) <u>Best response correspondences intersect:</u>

No player can profitably deviate given the strategy of the other players.

2) <u>Conjectures of the players are consistent:</u>

Each player i chooses s_i^* expecting all other players to choose s_{-i}^*.

BATTLE OF THE SEXES (BoS)

Two player game

(Two player of opposite sex M (Male) & F (Female).

F \ M	Opera	Football
Opera	3,2	0,0
Football	0,0	2,3

$$S_F = S_M = \{\text{Opera, Football}\}$$

$$S = S_F \times S_M$$

$$s \in S, \quad u_F(s) = u_F(s_F, s_M)$$
$$u_M(s) = u_M(s_F, s_M)$$

In the matrix, in each entry

(1) First number is the payoff to player 1 (row player, female)
Second number is the payoff to player 2 (column player, male).

(2) Player 1 chooses a row $s_F \in \{\text{opera, football}\}$
Player 2 chooses a column $s_M \in \{\text{Opera, football}\}$

(3) The payoffs are
$u_F(s_F, s_M)$, $u_M(s_F, s_M)$

GREEDY

Thus if F chooses opera and M chooses football then their payoffs are (0,0) (as they will not be able to enjoy each other's company.)
This is not a Nash equilibrium, since they can do better if either one deviates.

ULTRA - ALTRUISTIC.

If both of them are willing to make sacrifice for the other: F choosing football and M choosing opera then they also end up with utility :- (0, 0) $\not\equiv$ NASH EQ.

Note that if they choose strategies (opera, opera) ← M deviates from the greedy strategy.
then the pay-off increases from (0,0) in greedy strategy to (3, 2)
← F enjoys both the opera and M's company
M gains in utility by being in F's company.

$$u_F(\text{opera}, \text{opera}) > u_F(\text{football}, \text{opera})$$

$$u_M(\text{opera}, \text{opera}) > u_M(\text{opera}, \text{football})$$

Neither of them shoud deviate.
$\equiv$ NASH EQ.
(football, football) is another NASH EQ.

Ideal situation: If they can take turns to go to about equal number of (opera, opera) and (football, football).

SECOND PRICE AUCTION
(with Complete Information).

★ An object is to be assigned to a player $\{1, \dots, n\}$

★ Each player has his valuation of the object
v_i = Player i's valuation of the object.
$v_1 > v_2 > \cdots > v_n > 0$

★ Assume everyone knows all the valuations
$v_1, v_2, \dots, v_n$
{Complete Information Version}

- The players simultaneously submit bids $b_1, b_2, \dots, b_n$
- The object is assigned to the highest bidder (with random tie-breaking).
- The winner pays the SECOND HIGHEST BID.

The utility function

$$u_i(b_1, b_2, \dots, b_n) = \begin{cases} v_i - b_j & i\text{: highest bidder, } j = 2^{nd}\text{ highest bidder;} \\ 0 & \text{o.w.} \end{cases}$$

TRUTHFUL BIDDING.

Lemma.

In the second price auction, TRUTHFUL BIDDING, i.e.,

$b_i = v_i$ is a NASH EQ.

$\boxed{b^* = \langle v_1, v_2, \cdots, v_n \rangle}$ ← { TRUTHFUL EQUILIBRIUM.

Proof:

Player 1 receives the object and pays v_2.

$u_1(b^*) = v_1 - v_2$

$\forall_{j \neq 1} \; u_j(b^*) = 0$

Player 1 has no incentive to deviate, since

a) $b_1 < v_2$, decreases the pay-off to 0.

b) $b_1 > v_2$, has no effect. i.e. v_1 is as good as any other $b_1 > v_2$.

For player $j \neq 1$, $v_j < v_1$, So to change the payoff, he must bid $b_j > v_1 > v_j$ and receive a payoff of

$v_j - v_1 < 0$.

Player j has no incentive to deviate.. □

(Incompte Information Case) → Bit more complex.

- Two other Nash Eq.: $\begin{cases} \langle v_1, 0, \cdots, 0 \rangle \\ \langle v_2, v_1, 0, \cdots, 0 \rangle \end{cases}$
- Truthful bidding results in a Weakly Dominant Nash Equilibrium.