LOGIC
QUIZ #5
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Once again, we visit the Island of Knights and Knaves along with our Antropologist. In these islands, those called *knights* always tell the truth and *knaves* always lie. Furthermore, each inhabitant is either a knight or a knave.

Notation: $k =$ knight, $\neg k =$ knave.

Q1. [10] It was recently reported in the Times of Knights that O.J. Simpson, while visiting this island, got arrested and was put on trial. First the prosecutor pointed to the defendant and said: "If he is guilty, then the glove will fit." The defense attorney said: "That's not true." The judge did not know whether the defense or prosecuting lawyer is a knight or a knave, but with a little reasoning (and nothing else) figured out whether O.J. Simpson should be convicted or acquitted. What was the verdict?

Soln1. *Consider the following truth-table*:

Defense	Prosecutor's statement	Guilt
$\neg k$	T	$\top$ (T $\vee$ F)
k	F	T

In the first case (the defense lawyer is a knave), the statement "G $\to$ F $\equiv$ $\neg G \vee F$" can be true, no matter whether G is true or false. So, the judge could not have figured out whether O.J. Simpson is guilty or not.

In the second case (the defense lawyer is a knight), the statement "G $\to$ F $\equiv \neg(G \wedge \neg F)$" can be false iff G is true (and F is false). So, the judge figured out that O.J. Simpson is guilty, and that the defense lawyer is a knight.

Q2. [10] However the story was reported somewhat differently in the Knave Knews. In their version, O.J. Simpson, while visiting this island, got arrested and was put on trial. But, the prosecutor said something different: "If he is not guilty, then you must acquit. If he is guilty, then the glove must fit. So, if the glove does not fit, you must acquit." The defense attorney said: "That's not

true." The judge did not know whether the defense or prosecuting lawyer is a knight or a knave, but tried to figure out the answer with some reasoning (and nothing else). What was the verdict?

Soln2. *This is easier. What the prosecutor said is always true ($\top$ = tautology). First*

$$G \to F \equiv \neg F \to \neg G.$$

Thus
$$(\neg F \to \neg G) \wedge (\neg G \to A) \to (\neg F \to A).$$

You can show that the above is a tautology, using the Tableaux method. So the defense lawyer is a knave. There is no way to know if the defendant was guilty. (Perhaps, it ended in a mistrial.)