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Repeated games, Cheap talk

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see Gintis, *Game theory evolving*, Ch. 9, and mentioned papers

Finitely repeated games

- Intuition: repeated interaction may affect players' willingness to cooperate
- Finite repetition turns a stage game into a larger finite game
- Strategy: function that selects, at each stage, an action *given the history of play*
- For example, play 5 times Prisoner's dilemma in a row
- Examples for strategies:
  - Always cooperate
  - Cooperate twice and then always defect
  - Cooperate until opponent defects, then always defect (grim trigger strategy)
  - Cooperate in the first round, then always copy opponent's previous move (tit for tat)
  - ...
- How does repetition affect rational behavior / equilibria?

Finitely repeated Prisoner's dilemma

	C	D
C	2,2	0,3
D	3,0	1,1

↓

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- Backward induction still applies:
  - last round is normal PD, so both defect no matter what
  - so second last round has no effect on last round
  - so second last round is also normal PD
  - and so on until first round
- Makes no difference to the dilemma

Indefinitely repeated games

- In(de)finite repetition gets rid of backward induction
- There is no (known) end from which to start
- Allows to focus on long-term nature of interaction
- Can it enable cooperation?

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- We need to make sure that payoffs are well-defined
- Simple solution: discounting the future
- At any time, the value of tomorrow's subgame is multiplied by $0 < \delta < 1$
- Consider subgame at time t
- Value $v_i(t)$ to player i:

$$v_i(t) = \pi_i(t) + \delta v_i(t+1) = \pi_i(t) + \delta \pi_i(t+1) + \delta^2 \pi_i(t+2) + \dots$$

Trigger strategies allow cooperation in equilibrium

Theorem

Cooperation can be achieved as subgame perfect Nash equilibrium if δ is sufficiently close to 1.

Proof.

Consider both players using the grim trigger strategy. Then the value to each player at any time t is

$$2 + 2\delta + 2\delta^2 + \cdots = 2\frac{1}{1-\delta}$$

From defection, a player would obtain 3 once and then 0 forever. Thus, the strategies form an equilibrium if

$$2\frac{1}{1-\delta} \geq 3, \text{ or } \delta \geq \frac{1}{3} \qquad \square$$

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Tit for tat (Axelrod and Hamilton, 1981) famously allows cooperation in equilibrium as well (but not subgame perfect).

Folk theorem

Bad news: many equilibria, so not very informative

Theorem

Consider any two-player stage game with a Nash equilibrium with payoffs (a, b) *to the two players. Take any pair of strategies that gives payoffs* (c, d) *with* $c \geq a$ *and* $d \geq b$*. If* δ *is sufficiently close to* 1*, then there is a subgame perfect Nash equilibrium with expected payoffs* (c, d) *in each round.*

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- Punishment may incur cost for punisher
- For subgame perfection, punishment must be rational (credible threat)
- Punish failure to punish, failure to punish failure to punish, etc

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Good news: most equilibria are not evolutionarily stable

Outline

Repeated games

Cheap talk

Cheap talk

A verbal contract isn't worth the paper it's written on.
– Yogi Berra (or Samuel Goldwyn?)

- Cheap talk in a sense is a special case of signaling games
- But studied mostly in economics with a different perspective
- Often a pre-existing language is assumed
- Communication incurs no costs
- Communication is not necessarily truthful
- There is no way to enforce binding agreements

Thus, cheap talk is worthless.

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Thus, cheap talk is worthless?

No! There are situations in which cheap talk can be meaningful.

Under what conditions does cheap talk actually convey information, under what conditions is it credible?

Meaningful cheap talk

Joseph Farrell and Matthew Rabin. Cheap talk.
The Journal of Economic Perspectives, 10(3):103–118, 1996.

- Sally is applying for a job at Rayco
- There is a demanding open position and an undemanding one
- Sally's ability may be high or low
- Both benefit if she gets the appropriate job:
  - Sally will enjoy the demanding job iff she has high ability
  - Rayco will profit from giving her the demanding job iff she has high ability

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		Job Rayco gives Sally	
		Demanding	Undemanding
Sally's ability	High	$2, 1$	$0, 0$
	Low	$0, 0$	$1, 3$

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		Job Rayco gives Sally: Demanding	Job Rayco gives Sally: Undemanding
Sally's ability	High	$2, 1$	$0, 0$
	Low	$0, 0$	$1, 3$

When Sally tells Rayco her ability, Rayco can simply believe her: She has no incentive to lie.

Formalizing cheap talk – strategies and equilibrium

Cheap talk can be formalized as a two-stage game:

- First, Sally says something (usually about her type) to Rayco.
  A strategy for Sally thus maps her types to utterances.
- Then, Rayco chooses an action, in this case either to give her the undemanding job or the demanding one.
  A strategy for Rayco thus maps Sally's utterances to Rayco's actions.

A (Nash) equilibrium consists of a strategy for Sally and one for Rayco such that neither of them would benefit from changing their strategy.

This can be used as a first attempt to characterize outcomes of cheap talk games.

Formalizing cheap talk – equilibrium and self-signaling

For the example, we get this (truth-telling) equilibrium:

- Sally says "My ability is high" iff her ability is high
- Rayco gives her the demanding job iff she says "My ability is high"

This works because both of her types are self-signaling:

- she wants Rayco to believe she has high ability iff she has it
- she wants Rayco to believe she has low ability iff she has it

Talking about intentions

Assume Sender does not have a fixed type, but rather can choose an action (after talking).

		Calliope: Stag	Calliope: Rabbit
Artemis	Stag	$9, 9$	$0, 8$
	Rabbit	$8, 0$	$7, 7$

Artemis saying "I will hunt stag" is self-committing:
if Artemis thinks Calliope believes it, she will indeed hunt stag.

Self-signaling vs. self-committing

Sender's type/action X is

- self-signaling: Sender wants Receiver to believe that he is (going to do) X iff Sender indeed is;
- self-committing: if Sender believes that Receiver believes that Sender is going to do X, then Sender indeed is.

Aumann argues that "I will hunt stag" basically conveys no information because it is not self-signaling;
but that requires some intricate reasoning involving risk attitudes.

Robert J. Aumann. Nash equilibria are not self-enforcing.
In Jean Jaskold Gabszewicz, Jean-François Richard, and Laurence Wolsey, editors, *Economic Decision-Making: Games, Econometrics, and Optimization (Essays in Honor of Jacques Dreze)*, pages 201–206. Elsevier Science Publishers, Amsterdam, 1990.

Babbling example

Now assume that Sally wants the demanding (higher-paying) job even if the has low ability.

		Job Rayco gives Sally	
		Demanding	Undemanding
Sally's ability	High	$2, 1$	$0, 0$
	Low	$2, 0$	$1, 3$

Babbling example

Sally's preferences over Rayco's beliefs are no longer correlated with the truth:

- If she has high ability, she still wants Rayco to believe this;
- but now she wants Rayco to believe that she has high ability even if she has low ability!

Now, neither of her types is self-signaling:

- high ability is not because a low-ability Sally would also want Rayco to believe she has high ability;
- low ability is not because a low-ability Sally would not want Rayco to believe she has low ability.

No information will be conveyed; Sally might just as well babble randomly (unless she can engage in costly signaling, but that's a different story).

Preference alignment and credibility

In the two previous examples, Sally's preferences over Rayco's beliefs were either perfectly correlated with her true type, or not at all.

In a more general, continuous setting with partial correlation, it can be shown that imprecise cheap talk can convey some information.

Thus, cheap talk can

- convey full information if there is no incentive to lie;
- convey limited information if there is some incentive to lie;
- convey no information if there is a strong incentive to lie.

Vincent P. Crawford and Joel Sobel. Strategic information transmission. *Econometrica*, 50(6):1431–1451, November 1982.

A case against babbling equilibria

Consider again the first example:

		Job Rayco gives Sally: Demanding	Undemanding
Sally's ability	High	$2, 1$	$0, 0$
	Low	$0, 0$	$1, 3$

Intuitively, the truth-telling equilibrium should be the outcome. Unfortunately, there is also a babbling equilibrium:

No matter what Sally says, Rayco always gives her the undemanding job; Sally simply says "blubb".
Neither of them can improve by changing their own strategy (given a prior 50-50 belief of Rayco).

Still, this situation is unlikely to occur in reality. We thus need a criterion to find the "right" equilibrium.

Maybe the most informative equilibrium is the solution?

A case against full-information equilibria

		Rayco gives Sally		
		Job1	Job2	Manager
Sally can do	Job1	$2, \quad 5$	$1, -2$	$3, 4$
	Job2	$1, -2$	$2, \quad 5$	$3, 4$

Truth-telling and believing is a full-information equilibrium. This is counterintuitive, since Sally would prefer to be Manager, and Rayco a priori would give her that job.

A case against full-information equilibria

		Rayco gives Sally: Job1	Job2	Manager
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Truth-telling and believing is a full-information equilibrium. This is counterintuitive, since Sally would prefer to be Manager, and Rayco a priori would give her that job.

This can be justified if all statements are interpreted to mean either Job1 or Job2, so saying "no comment" is not possible.

This in turn is justifiable if according statements are simply not available. Even though a shared pre-existing language is assumed, it is not a rich language.

Neologisms

Joseph Farrell.
Meaning and credibility in Cheap-Talk games.
Games and Economic Behavior, 5(4):514–531, October 1993.

$$
\begin{tikzcd}
t_1 \arrow[r] & s_1 \arrow[r] & a_1 \\
t_2 \arrow[ur] & s_2 \arrow[ur] & a_2 \\
t_3 \arrow[uur] & s_3 \arrow[uur] & a_3
\end{tikzcd}
$$

- Equilibria often require that out-of-equilibrium (unused) signals are meaningless
- If we assume that there is some sufficiently expressive pre-existing language, such equilibria break down
- A neologism is an unused but meaningful signal
- For example, the natural language sentence “I’m type t_3”
- Credibility: similar as before
- An equilibrium is neologism-proof if, even in the presence of meaningful neologisms, using them yields no advantage
- Related to deception and evolutionary arms races
- In cells, we may consider “hardwired” or evolutionarily older meanings, with conventional or younger meanings evolving