Biology X, Fall 2010

Some more game theory and signaling games

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see Gintis, *Game theory evolving*

Overview

More on solution concepts

- Nash's theorem
- Equilibrium refinements

Signaling games

Solution concepts

A lot of game theory is about solution concepts.

- Given a game, how would/should players act?
- Usually circular reasoning
- Solution concepts provide a way out
- Many stability / equilibrium concepts capturing various intuitions

Basic important issues:

- Existence: is there always guaranteed to be one?
- Equilibrium selection: which one is most reasonable, if there is more than one?

Nash's theorem

Theorem

Every finite game has a Nash equilibrium in mixed strategies.

Proof.

Using a kind of best-response dynamics:

For any mixed strategy σ and player i, let

$$b_i(\sigma_{-i}) = \{\sigma_i' \mid \sigma_i' \text{ is a best response to } \sigma_{-i}\}$$

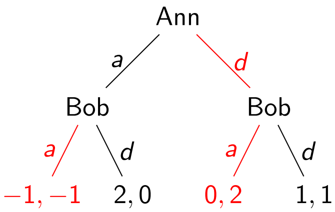
and let

$$b(\sigma) = b_1(\sigma_{-1}) \times \cdots \times b_n(\sigma_{-n}).$$

By definition, any σ with $\sigma \in b(\sigma)$ is a Nash equilibrium.

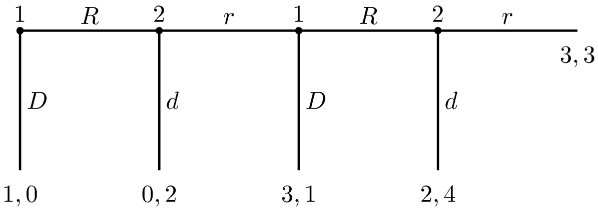
By Kakutani's fixed point theorem, such a σ exists. $\square$

Subgame perfect equilibria



- incredible threat
- subgame perfect equilibrium requires Nash equilibrium at all subtrees
- different with pre-commitment

Backward induction



- Centipede game
- Backward induction gives subgame perfect equilibrium
- Lots of philosophical discussion about rationality

Evolutionarily stable strategy

- Consider population repeatedly playing a stage game
- Stage game is symmetric in strategies and payoffs

- Assume σ reflects the current state of the population, τ a small mutant population
- Idea: population is stable if it cannot be invaded by mutants

- σ is an Evolutionarily Stable Strategy (ESS) iff
  - $\pi(\sigma, \sigma) > \pi(\tau, \sigma)$
    (σ is better in most encounters)
  - or $\pi(\sigma, \sigma) = \pi(\tau, \sigma)$ and $\pi(\sigma, \tau) > \pi(\tau, \tau)$
    (σ is as good as τ in most, but better in rare encounters

Trembling hand perfect equilibrium

Bob

		L	R
Ann	U	$1, 1$	$2, 0$
	D	$0, 2$	$2, 2$

- Two pure-strategy Nash equilibria: (U, L), (D, R)
- Only (U, L) is stable against small "mistakes" (trembling hand perfect)
- (D, R) gives more payoff but is more "risky"

Outline

More on solution concepts

- Nash's theorem
- Equilibrium refinements

Signaling games

Signaling phenomena

- Gazelles jump vertically when they see a cheetah
- Vervet monkeys have alarm calls for different predators
- Employees use degrees to signal their education

Gazelle and cheetah

- Gazelle has two types: Fit, Sick
- Gazelle has two signals: Jump, Don't jump
- Cheetah has two actions: Chase, Leave

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- Coevolution establishes an equilibrium in which information is transmitted

Signaling games

- Sender has a “type” (state, private information)
- Sender chooses a signal
- Receiver responds by choosing an action
- Payoffs depend on type and action (and signal)

- A sender strategy maps types to signals
- A receiver strategy maps signals to actions
- An equilibrium is a pair of strategies such that neither can improve by deviating

How can this be applied to intercell signaling?

- Cells are both senders and receivers
- What are the states and the actions?
- What are the payoffs?
- How about coevolution in a uniclonal (multicellular) setting?
- ...