Puzzle Corner

By Allan J. Gottlieb, '67

Once again I should like to extend my thanks to the many readers who have submitted problems and solutions. My friends are forever envious of my full mailbox. Several of my neighbors—in particular John Ross, '67, and Joel Shwimer, '67—have requested, "Would all those girls who send you solutions please enclose a recent photograph." John is particularly interested in Anne Cohn from Michigan.

Keep sending your problems and solutions to me. Address them to Baker 4380, 362 Memorial Drive, Cambridge, Massachusetts 02139.

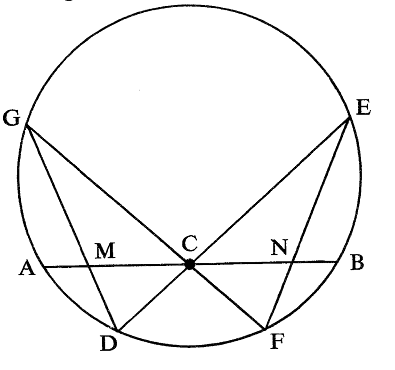
Problems

28♦—Mike Shupp, a former general manager of *TEN*, has posed the following problem:

1 is the smallest integer that is a perfect cube, a perfect 4th power, and a perfect 5th power. What is the next smallest integer with this property? Any number raised to the 60th power has the property, but surely there is some number smaller than 2^{60}. But I haven't found it yet.

29♦—Milton E. MacGregor, '07, submitted the following geometrical puzzle:

Given a circle, Q, with chord AB and C its midpoint. Through C draw any two chords DE and FG. Form DG and FE intersecting AB at M and N respectively.



Prove MC = NC.

30—Here's one from Paul W. Abrahams, '56:

Arrange 10 dots on a piece of paper *("10 points in* [0,1] x [0,1]*" for you mathy types—ed.)* in such a way that five line segments can be drawn through the dots, and each line segment contains exactly four dots. No two line segments can be colinear.

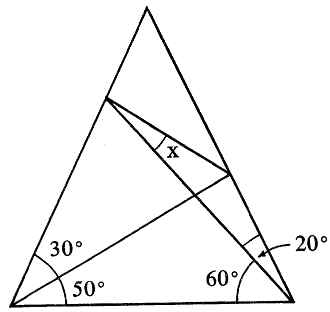
Let's see some cool shapes.

31♦—I have been told by several freshmen that my problems are too easy. Doesn't that fit the "M.I.T. Freshman" image perfectly? For their benefit (and for any new industrious readers), I shall reprint a problem which no one solved when it appeared last year in *TEN*. As an added inducement, the first person to submit a correct answer will receive a year's subscription to *TEN* or Technology Review. This problem was submitted by "the little guy," Harvey Friedman (his class varies as the 4th power of the current humidity).

Let C be the set of those reals whose base 3 expansions have no 1's. Can every real be expressed as a finite sum of elements of C?

OK, frosh, consider yourselves challenged.

32—Roger W. O'Dell, '68, wants you to find x:



33—Since I have such a lovely backlog of problems (keep 'em coming) I will dispense with the *Speed Department* and print two more regular problems. The first is from David W. Drumm:

What is the least number of queens that can be placed on a chessboard in a manner such that any additional queen would result in three being lined up?

Dave advises me to print any solution with fewer than 14 queens. So be it.

34—The final problem is from Richard Bator, '65:

You sit south and must make 7 spades. West's opening lead is the king of clubs. How do you play the hand given the following distribution?

North (dummy)
♠ A K Q J
♥ A Q 8
♦ Q J 10 9 8 7
♣ —

West
♠ 5 4 3 2
♥ K J 10 9 7 6
♦ —
♣ K Q J

East
Immaterial

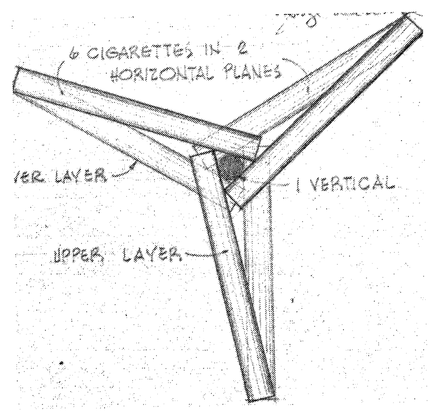
South (declarer)
♠ 10 9 8 7 6
♥ —
♦ A K
♣ A 10 9 8 7 6

At first it looks like you have it off the top. But when you start playing you realize that all vital transportation is missing. Then you think there is no solution. But there is!

Solutions

15—Place 7 unbent cigarettes such that each one is touching the other 6.

The Green Phantom sent in a beautiful full-scale model. Thanks, G.F., your Gauloises masterpiece is now one of my prized possessions. A drawing came from Eric Rosenthal, and George L. Downie, '51, sent the drawing reproduced here:

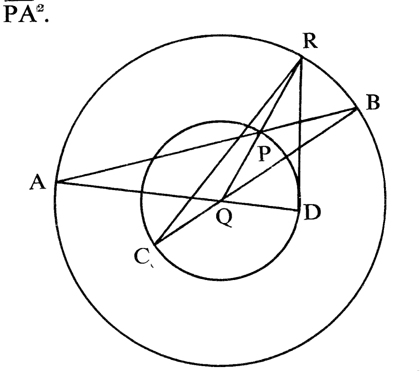


This problem was also solved by Murray E. Denofsky of Project MAC.

16♦—Prove that none of the Platonic solids may be dissected into another (two-page proof).

Mark H. Yu, '70, sent in a published solution.

17♦—Prove that $\overline{RD}^2 - \overline{RC}^2 = \overline{PB}^2 - \overline{PA}^2$.



The circles have center Q, and CB and AD intersect at Q.

Douglas Hoylman, '64, sent in the following solution.

Let r = small radius = CQ = PQ = DQ
Let s = large radius = AQ = RQ = BQ
Let w = angle RQD
Let x = angle RQC
Let y = angle PQB
Let z = angle PQA
Applying the law of cosines:
to triangle RQD: $\overline{RD}^2 = r^2 + s^2 - 2rs \cos w$
to triangle RQC: $\overline{RC}^2 = r^2 + s^2 - 2rs \cos x$
to triangle PQB: $\overline{PB}^2 = r^2 + s^2 - 2rs \cos y$
to triangle PQA: $\overline{PA}^2 = r^2 + s^2 - 2rs \cos z$
Hence $\overline{RD}^2 - \overline{RC}^2 = 2rs (\cos x - \cos w)$
and $\overline{PB}^2 - \overline{PA}^2 = 2rs (\cos z - \cos y)$.
But $w + z = x + y = \pi$;
hence $\cos w = -\cos z$ and $\cos x = -\cos y$,
so $\cos x - \cos w = \cos z - \cos y$;
therefore $\overline{RD}^2 - \overline{RC}^2 = \overline{PB}^2 - \overline{PA}^2$.

Mark Yu and Claire Parkinson (Wellesley '70) also sent in solutions, and I have a beautiful and comprehensive proof from Helen Means.

18♦—Evaluate in five seconds or less:

Puzzle Corner

$$\int_{-e^{-e^{2}}}^{\sin^3 4} \int_{\int_{-2}^{2}(y^3+3)\,dy}^{25\cosh(\ln 3)} \int_{-e^{-\ln(1/2)}}^{\int_{\pi}^{2\pi}\sin X\,dX} \frac{(\cosh^2(x+2)\sin x^3)^{x^2-x}\,dx}{\ln(e^{2x}+\ln 2x^{e^x}-1)^{x+2\sin x}}$$

Alas, this one slipped by the "old pro" himself. Yes, Virginia, problem 18 was trivial with 0 the obvious answer. As expected, the insults poured in. Here are a few:

Dear Mr. Gottlieb:

Did you really take my problem seriously? How many readers noticed that even though I wrote a triple integral, I only wrote one differential to integrate over? That was my mistake.

Once that is corrected, there are two solutions to the problem. The classic solution that any 18.01 dropout can see is that

$$\int_{\pi}^{2\pi} \sin x\,dx = -e^{-\ln 1/2} = -2,$$

from which it can be seen that the integral must be 0. A new solution has just been brought to my attention: Any integral of that complexity must equal 0, because no one would have the patience to actually work something like that.

Mark B. Pelcovits, '70
East Campus

Dear Mr. Gottlieb:

I suggest that the problem was submitted as a test more of *your* intelligence than the reader's. Since one quickly notices:

$$-e^{-\ln 1/2} = \int_{\pi}^{2\pi} \sin x\,dx = 2$$

everything disappears and = 0 is answer.

Hereafter publish only useful problems, please.

Charles Dorian, '68
Student House

First of all the limits are *not* equal to 2, as Mr. Pelcovits has pointed out. Second, why don't you *answer* only useful problems.

Also solved by Leo G. Chouninard, '70, David W. Thiel, '70, Robert A. Parker, '70, Kyoichi Haruta, '63, Mark H. Yu, '70, Paul Schweitzer, '61, and Miss Parkinson.

22—Let (a_α) be a non-empty set of reals. Define the distance set of (a_α) to be (b such that $b = a_\beta - a_\gamma$). What can be said about distance sets (measure, open, closed, connected, etc.)? In particular, what are some necessary and/or sufficient conditions for a set A to be the distance set of any set. For example, A must contain 0 and A cannot be (0,1,3).

Despite the fact that I offered a free subscription to *Tech Engineering News* for the best solution to this problem, I did not receive any solutions. Now the offer is extended, and the prize becomes the reader's choice of free subscriptions to *TEN* or Technology Review.

Mr. Hoylman, who is now going for a Ph.D. in mathematics at the University of Arizona, writes:

Dear Mr. Gottlieb:
The distance set puzzle as given in the November Technology Review is the sort of thing that I can't stop working on once I get started. I was able to prove the following results: (dA is the distance set of A).

If A is open, connected, bounded, or compact, then so is dA, but if A is closed, dA need not be closed.

If A is infinite, the cardinality of dA is equal to that of A.

If A has n elements and dA has m elements, then $2n - 1 \leq m \leq n(n - 1) + 1$, and these are the best possible bounds.

$A = dA$ if A is a subgroup of the additive group of real numbers.

There is a set of Lebesgue measure zero whose distance set is the entire real line.

You probably are already familiar with most of these properties, but if any are new to you let me know and I'll write up a proof. The one thing I did not succeed in doing was to find a necessary and sufficient condition that a set be the distance set of some set (though there is the necessary, but very insufficient, condition that the set contain 0 and be symmetric about it, and the sufficient, but quite unnecessary, condition that the set be a subgroup of the reals). From the above results it doesn't seem possible to characterize it by cardinality, measure, or topological or algebraic properties.

Thank you, Doug, and good luck at Arizona. This problem first appeared in *Tech Engineering News* for April, 1966, and found no solution. We reprinted it again this fall, offering a free subscription to either *Tech Engineering News* or Technology Review for a solution. So let me know if you want your free one-year subscription to be for *TEN* or TR. I am sorry but I have no similar arrangements with *Playboy*. I would be very happy to receive your proofs, and will print all that space permits.

Due to deadline variations and space limitations over which I have no control, many readers did not receive credit for successful solutions. Although I doubt that anyone really would be heartbroken if his name did not appear, I consider myself morally obliged to give credit where credit is due.

These people sent in solutions to the problems indicated:

1. Douglas K. Severn, '23, Douglas King, and Elizabeth Cox Worden, daughter of Edwin S. Worden, '31.
2. David W. Drumm, '68, Martin Landau, Frank R. Hanawalts, '54, (Pennsylvania State University), Steven Raneri, Mr. Severn, and Mr. King.
4. Douglas Hoylman, '64, Mr. Severn, Mr. King, and Mr. Landau.
5. George Schnitzler, '21, Milton E. MacGregor, '07, Mr. King, and Mr. Severn.
6. Mr. King and Mr. Severn.
7. Mr. Landau, Mr. King, and Mr. Severn.
8. Robert G. McKean, '64, and Mr. Hoylman.
9. Mr. McKean, Mr. Hoylman, Mr. Landau, and Mr. Severn.
10. John Joseph, David C. Lukens, '57, Mr. McKean, Mr. Landau, and Mr. Severn.
11. Captain John Woolston, '44, Mr. McKean, Mr. Landau, and Mr. Hoylman.
12. Captain Woolston, Mr. McKean, Mr. Landau, Mr. Hoylman, Mr. Joseph, Mr. Drumm, Mr. Severn, Hubert duB. Lewis, '37, Gardner Perry 3rd, '61, Mark Pfeil, John Newbegin, '34, Eugene F. Kelly, '54, Joseph N. Feil, '60, Stanley A. Horowitz, '66, Anne Harris Cohn, '67 (University of Michigan), Alexis Ostapenko, Martha C. Morrison (daughter of Samuel G. Morrison, '44), Russell A. Nahigian, '57, Alexander F. Robb, '41, Paul Siegel (son of Professor Abraham J. Siegel), John Fielding, Donald F. deRegnier, '60, Earl J. Rogers, '59, and Richard Bator, '65.
13. Mr. Joseph, Mr. Bator, Mr. McKean, Mr. Hoylman, and Mr. Severn.
14. Frank G. Smith, '11, Thomas S. Fulmer, '61, Captain Woolston, Mr. Joseph, Mr. McKean, Mr. Hoylman, Mr. deRegnier, Mr. Feil, and Miss Morrison.

Letters on Review

Prime Mover

To the Editor:

The article on Sensory Aids in the December Review is excellent—as history and as a résumé of current work.

I hope you'll pardon a bit of personal pique, if, as the prime mover now six years of the Mechanical Engineering Sensory Aids group, the proposer, first year principal investigator, and currently Steering Committee Chairman of the Center for Sensory Aids Evaluation and Development, and Chairman of the Subcommittee on Sensory Aids of the National Academy of Sciences-National Research Council, I felt somewhat left out.

ROBERT W. MANN, '50
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The apologies are all due from Technology Review.—Ed.