

# Logic and Verification – due April 1, 2003

1. Give a deduction (from  $\emptyset$ ) of  $(\forall x \phi) \rightarrow \exists x \phi$ .

2. (Re-replacement lemma)

(a) (a) Show by example that  $(\phi_y^x)_x^y$  is not in general equal to  $\phi$ . Show that it is possible both for  $x$  to occur in  $(\phi_y^x)_x^y$  at a place where it does not occur in  $\phi$ , and for  $x$  to occur in  $\phi$  at a place where it does not occur in  $(\phi_y^x)_x^y$ .

not occur at all in  $\phi$ , then  $x$  is substitutable for  $y$  in  $\phi_y^x$  and  $(\phi_y^x)_x^y = \phi$ .  
deduction on  $\phi$ .

property of equality follows from the axioms, that is,  $\vdash \forall x \forall y \forall z (x = y \rightarrow$

steps 3 and 4 are valid.

of the following two statements.

$\phi$ .

of formulas is satisfiable.

$\{Pv_1, Pv_2, Pv_3, \dots\}$ . Is  $\Gamma$  consistent? Is  $\Gamma$  satisfiable?