Recursion

Something that defines, describes, or is composed using itself.
Recursion

- A recursive definition
Recursion

• A recursive definition
  • Recursion
    – See “Recursion”
Recursion Examples

• Fractals
  • Sierpinski Triangle
    - 1: start with a triangle
    - 2: divide into 4 triangles
    - 3: for each corner, go back to step 1.
Recursion Examples

- Fractals
  - Sierpinski Triangle
  - Koch's Snowflake
    - 1: Start with a triangle
    - 2: in the middle of each edge, make a new triangular bump
    - 3: repeat step 2 for each new bump
Recursion in Imagery

Source: xkcd.com
Recursion in Imagery

Source: wikipedia.org
Basic Daily Examples
Basic Daily Examples

• Calculating Compounded Interest
  • Interest rate of 3% compounded yearly:
    – $1000 after 1 year: $100*1.03 = $103
    – $1000 after 2 years ($100*1.03)*1.03 = $106.09
    – $1000 after 3 years (($100*1.03)*1.03)*1.03 = $109.2727
Basic Daily Examples

• Calculating Compounded Interest
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• Population Growth
  • Population growth of 1.1% per year:
    - 6.7Billion*1.011 = 6.774 Billion
    - (6.7Billion*1.011)*1.011 = 6.848 Billion
Biological Examples

- Romanesco
- Broccoli
Biological Examples

- Broccoli
- Cell Replication (Mitosis)
Mathematical Examples

Factorial:

• $1! == 1$
• $2! == 2 * 1!$
• $3! == 3 * 2!$
• $4! == 4 * 3!$
• ...
• $n! == n * (n-1)!$
Mathematical Examples

Fibonacci Series:

- 1, 1, 2, 3, 5, 8, ...
- fib(1) == 1
- fib(2) == 1
- fib(3) == fib(1) + fib(2)
- fib(4) == fib(2) + fib(3)
- ...
- fib(n) == fib(n-2) + fib(n-1)
Recursion in Meme

YO DAWG I HERD YOU LIKE CARS SO WE PUT A CAR IN YO CAR SO YOU CAN DRIVE WHILE U DRIVE
Recursion in Meme

\[ \frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \]

YO DAWG I HERD YOU LIKE FUNCTIONS SO WE PUT A FUNCTION IN YO FUNCTION SO YOU CAN DERIVE WHILE YOU DERIVE
Recursion in Meme

Yo dawg, we heard you like recursion, so we thought...
Recursion in Meme

Yo dawg i heard you like dreams.

So i put a dream in yo dream, so you can dream while you dream.
Recursion in Computer Science

An algorithm that uses itself to solve a problem is a recursive algorithm.
Recursive Functions

function(params):
  # Case 1:
    # Base Case: check for termination
  # Case 2:
    # Recursive call, progressing towards base case
Base Case

What happens if you don't have a base case?
Base Case

What happens if you don't have a base case?
Infinite recursion.
Base Case

What happens if you don't have a base case?

Infinite recursion

• Examples of infinite recursion:
Base Case

What happens if you don't have a base case?

Infinite recursion

- Examples of infinite recursion:
  - Fractals
  -
Base Case

What happens if you don't have a base case?

Infinite recursion

- Examples of infinite recursion:
  - Fractals
  - Cancer (uncontrolled cell replication)
Factorial Recursive Function

Example:

```python
factorial(n):
    if n == 0: # base case
        return 1
    else: # progress towards the base case
        return n * factorial(n-1)
```

• See Example code.
Fibonacci

See example code.
Zeno Paradoxes

If going from point A to point B and we move half the distance between our current point (starting at point A) and point B, how many times will it take us to reach point B?
Towers of Hanoi

- 3 Pegs: A, B, C
- We have N disks that go on the pegs in increasing sizes
- We label this disk from smallest to largest as disks 1 - N
- We can move 1 disk at a time
- No disk can be placed on top of a disk that is smaller than itself (disks can only be in a pyramid style stack)
Towers of Hanoi - Solution

- To move $n$ discs from peg A to peg C:
  1. move $n-1$ discs from A to B. This leaves disc #$n$ alone on peg A
  2. move disc #$n$ from A to C
  3. move $n-1$ discs from B to C so they sit on disc #$n$