POKER

The number of hands is $\binom{52}{5}$, read "52 choose 5," found by doing ${}_{52}C_5$ on your calculator. This number is $2,598,960$ and is the denominator when we do the probabilities below.

• Royal Flush. There are 4 of these, one in each suit. Probability is $\frac{4}{2{,}598{,}960}$. Since it is so small we work with the reciprical which is 649740. So the chances are one in 649740.

• Straight Flush. There are four possible suits and in each suit eight possible straight flushes – one way to see this is that the top card must be from $6, 7, 8, 9, 10, J, Q, K$ – so 32. The probability s $\frac{4}{2{,}598{,}960}$. Since it is so small we work with the reciprical which is 81000 and change. So the chances are roughly one in 80000. Also observe that it is eight times more likely than a royal flush.

• Four of a Kind. (e.g.: $3\heartsuit, 3\clubsuit, 3\diamondsuit, 3\spadesuit, 7\heartsuit$) There are 13 choices for the rank (here 3) and 48 choices for the other card (here $7\heartsuit$) so $13 \cdot 48 = 624$ possibilities. Probability is $\frac{624}{2{,}598{,}960}$. Since it is so small we work with the reciprical which is about 4165. So the chances are about one in 4165.

• Full House. (e.g.: $7\heartsuit, 7\clubsuit, 7\diamondsuit, J\spadesuit, J\heartsuit$). There are 13 choices for the rank of the trips (here 7) and then 12 choices for the rank of the pair (here J) and $\binom{4}{3} = 4$ choices for the suits of the trips (here $\heartsuit, \clubsuit, \diamondsuit$) and $\binom{4}{2} = 6$ choices for the suits of the piar (here $\spadesuit, \heartsuit$) so the total number is $13 \cdot 12 \cdot 4 \cdot 6 = 3744$. Probability is $\frac{3744}{2{,}598{,}960}$. Since it is so small we work with the reciprical which is about 694. So the chances are roughly one in 700.

• Flush (e.g. $3\heartsuit, 8\heartsuit, J\heartsuit, K\heartsuit, A\heartsuit$). There are four choices for the suit ($\heartsuit$) and then $\binom{13}{5} = 1287$ for the ranks $(3, 8, J, K, A)$ so $4 \cdots 1287 = 5148$. *But* 32 of those are straight flushes and 4 of those are royal flushes. Taking those away there are 5112 (ordinary) flushes. Probability is $\frac{5122}{2{,}598{,}960} \sim 0.0019$. Since it is so small we work with the reciprical which is about 508. So the chances are roughly one in 500. Rarer than a full house, but not by much.

• Straight (e.g. $7\heartsuit, 8\clubsuit, 9\diamondsuit, 10\heartsuit, J\heartsuit$) There are nine choices for the top card rank. (It can't be $2, 3, 4, 5$ and can be $6, 7, 8, 9, 10, J, Q, K, A$.) Then each suit has four possibilities. So this gives $9 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 9216$. Again we need substract the 36 straight and royal flushes so there are 9180 that are ordinary straights.Probability is $\frac{9216}{2{,}598{,}960} \sim 0.0035$. Since it is so small we work with the reciprical which is about 282. So the chances are roughly one in 280. As good poker players know, straights are *considerably* more likely than flushes, nearly twice as likely.

• Three of a Kind. (e.g.: $7\heartsuit, 7\clubsuit, 7\spadesuit, J\diamondsuit, 4\heartsuit$) There are 13 choices for the rank of the trips, here 7. Then there are $\binom{4}{3} = 4$ choices for the suits of the trips, here $\heartsuit, \clubsuit, \spadesuit$. The other two cards must be chosen from the cards not of that rank (here, they can't be 7s) so they have 48 cards to choose from and they have $\binom{48}{2} = 1128$ possibilities which gives $13 \cdot 4 \cdot 1128 = 58656$. We need subtract the 3744 full houses so that there are 54912 (ordinary) three of a kind hands. Probability is $\frac{54912}{2{,}598{,}960} \sim 0.0211$. That is, around 2% or, taking reciprocals, about one in 47.

• Two Pair. (e.g.: $9\diamondsuit, 9\clubsuit, 4\heartsuit, 4\clubsuit, Q\spadesuit$). There is a subtlety here. There are $\binom{13}{2} = 78$ choices for the two ranks $9, 4$. We count combinations because the order does not count. Nines and Fours is the same as Fours and Nines. Note that this is not the case with a full house, Nines and Fours, Nines Up is not the same as Nines and Fours, Fours Up. Now there are $\binom{4}{2} = 6$ choices for the suits of the first pair $(\diamondsuit, clb)$ and $\binom{4}{2} = 6$ choices for the suits of the second pair $(\heartsuit, clb)$ and $52 - 8 = 44$ choices for the fifth card $(Q\spadesuit)$ since it can't be in either ranks of the pairs so $78 \cdot 6 \cdot 6 \cdot 44 = 123552$.Probability is $\frac{123552}{2{,}598{,}960} \sim 0.0475$. That is, around 5% or, taking reciprocals, about one in 21.

• One Pair. (e.g.: $8\diamondsuit, 8\heartsuit, Q\spadesuit, 9\spadesuit, 6\clubsuit$). There are 13 choices for the rank of the pair (here 8) and $\binom{4}{2} = 6$ choices fo the suits of the pair (here $\diamondsuit, \heartsuit$). Now here is a clever counting way that avoids worrying about better hands. There are $\binom{12}{3} = 220$ choices for the other three ranks (here $Q, 9, 6$). (They must be all different as otherwise you'd have two pair or better.) Then there are 4 choices for the suit for each of these cards (here $\spadesuit, \spadesuit, \clubsuit$ respectively) so the total is $13 \cdot 6 \cdot 220 \cdot 4 \cdot 4 \cdot 4 = 1{,}098{,}240$. Probability is $\frac{1098240}{2{,}598{,}960} \sim 0.42$. That is, around 42%. *Very* roughly, somewhat less than half the time.