Basic Algorithms, Assignment 10

Due, TUESDAY, Nov 20

1. A positive integer n is called *banana* if it can be written as the sum of two integer squares. (E.g., $41 = 16 + 25$.) Argue that the problem class BANANA is in NP.

2. In 2002 Agarwal, Kayal and Saxena showed that PRIME is in P. Call a positive integer n *walrus* if it is the product of precisely three primes. Given the AKS result, argue that WALRUS is in NP. Argue (harder!) that NOTWALRUS is in NP.

3. Lets define `3-SATSMALL` to be the same as `3-SAT` except that no Boolean variable x_i appears (as either x_i or $\overline{x_i}$) more than 20 times. Our object is to show `3-SAT`$\leq_P$`3-SATSMALL`.

   (a) Let x, y be Boolean variables. Find a set of clauses $C_1, \ldots, C_r$ of size 3 using auxilliary Boolean variables $z_1, z_2, \ldots z_s$ so that $C_1 \wedge \ldots \wedge C_r$ can be satisfied if and only if x, y have the same truth value. (Here r, s will be small numbers.)

   (b) Let $x_1, \ldots, x_w$ be Boolean variables. Show that there is a set of clauses $C_1, \ldots, C_v$ using auxilliary variables $z_1, \ldots$ so that $C_1 \wedge \ldots \wedge C_v$ can be satisfied if and only if $x_1, \ldots, x_w$ have the same truth value. Further, we require that none of the Boolean variables (neither the original x's nor the auxilliary z's) are used more than ten times. (Idea: Make sure x_i, x_{i+1} have the same truth value for $1 \leq i < w$.)

   (c) Show `3-SAT`$\leq_P$`3-SATSMALL`. (Idea: When x_i appears many times replace it with copies $x_i^1, x_i^2, \ldots$, none appearing very often, that all must have the same truth value.)

4. Suppose a graph algorithm with input a graph G takes a time polynomial in $N+M$ where N is the number of vertices and M is the number of edges in G. Show that it takes time polynomial in N. Suppose a number theoretic algorithm with input a positive integer x takes time polynomial in x. What can you say about the time it takes when the input is an n-digit number? In particular, explain why you *cannot* say that the time is polynomial in n.

What I tell you three times is true
– Lewis Carroll *in* Hunting the Snark