HOME WORK 1: THEORY OF COMPUTATION
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1 Logic

Problem. 1
It is year 2001! It has been TEOTWAWKI (the end of the world as we knew it). The Y2K bug has brought an end to the human civilization. Fortunately, the computers are not dead! There are two kinds of computers that have survived a Darwinian struggle: Bill's computers (manufactured by William "Bill" Gates of Microsoft) and Gill's computers (manufactured by Guglielmo "Gill" Bates of Megahard). These machines are claimed to give correct answers to any yes/no questions instantly. The output is given by flashing a green or a red light—one for yes, the other for no; unfortunately, it is not known which one is which.

Bill's and Gill's computers have identical appearance, but they differ in one aspect, they use different colors for the yes/no answers. For instance, if Bill's computers use green for yes then Gill's computers use red for yes and vice versa.

Your job is the following:

1. You are shown a machine. You are allowed to ask just one question to find out whether it is a Bill's machine or a Gill's machine. What question would you ask?
2. A machine is called "natural" if it uses green for yes. Your job is to just ask one question to determine which kind of machine is natural. What question would you ask?
3. You are shown a machine. You will get it for free and fifty cows, if you can ask it one yes/no question that it cannot answer. What question would you ask?

2 Finite Automata

Problem. 2

1. Exercise 1.15 (Sipser, pp 86).
2. Exercise 1.25 (Sipser, pp 88).

3 Pumping Lemma

Problem. 3

1. Consider the language consisting of the set of all strings with sequence of 0's followed by the same number of 1's.

$$\mathcal{L} = \{o^n 1^n | n \geq 0\}.$$

   Show that it's not regular.

2. Using a homomorphism reduction show that the set of palindromes PAL(Σ) over $\Sigma = \{0, 1\}$ is not regular either. (DO NOT USE THE PUMPING LEMMA DIRECTLY.)

$$\text{PAL}(\{0,1\}) = \{\sigma \in \{0,1\}^* | \sigma = \sigma^R\}.$$

3. Consider the set of reverse palindromes RPAL(Σ) over $\Sigma = \{A, T, C, G\}$, defined as follows: The complementation operation is a homomorphism defined by $A^C = T$, $T^C = A$, $C^C = G$, and $G^C = C$. Show that the language RPAL(Σ) is not regular. (Again, DO NOT USE THE PUMPING LEMMA DIRECTLY.)

$$\text{RPAL}(\{A,T,C,G\}) = \{\sigma \in \{A,T,C,G\}^* | \sigma^R = \sigma^C\}.$$

4 Miscellaneous [HARD]

Problem. 4 Let $\mathcal{L} \subseteq 0^*$. Prove that $\mathcal{L}^*$ is regular.