Biology X, Fall 2010
Signaling games

Andreas Witzel

CIMS, NYU

2010-11-01

see Skyrms, *Signals* and Gintis, *Game theory evolving*
Vervet monkeys

- Vervet monkeys have distinct alarm calls for different predators
  - Eagle: “cough” ➞ hide in the underbush
  - Leopard: “bark” ➞ climb on a tree
  - Snake: “chutter” ➞ watch out for snake

- Even inter-species communication exists
- How can such systems come about?
- How can meaning evolve?
- Can we give an explanation that is simple enough to apply even to bacteria and cells?
Sender-receiver games

- Introduced by David Lewis (1969) to explain convention and meaning
- “Worst-case scenario” in which natural salience is absent and signaling is purely conventional
- Two players: sender, receiver
- Sender has a “type” (state, private information)
- Sender chooses a signal (signals have no intrinsic meaning)
- Receiver responds by choosing an action
- Payoffs depend on type and action (and signal)
- A sender strategy maps types to signals
- A receiver strategy maps signals to actions
- An equilibrium is a pair of strategies such that neither can improve by deviating
Basic definitions

- Set of types $T$, signals $S$, actions $A$
- Probability distribution $\tau \in \Delta T$
- Sender strategy $\sigma : T \rightarrow \Delta S$
- Receiver strategy $\rho : S \rightarrow \Delta A$
- Payoff for sender: $u(t, s, a)$, for receiver: $v(t, s, a)$
- Equilibrium: pair of strategies $\sigma, \rho$ such that

$$\sum_{t, s, a} u(t, s, a) \tau(t) \sigma(s \mid t) \rho(a \mid s) \geq \sum_{t, s, a} u(t, s, a) \tau(t) \sigma'(s \mid t) \rho(a \mid s)$$

and

$$\sum_{t, s, a} u(t, s, a) \tau(t) \sigma(s \mid t) \rho(a \mid s) \geq \sum_{t, s, a} u(t, s, a) \tau(t) \sigma(s \mid t) \rho'(a \mid s)$$

for all $\sigma', \rho'$
Simple example

<table>
<thead>
<tr>
<th>Sender type</th>
<th>Receiver action</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a₁</td>
</tr>
<tr>
<td>t₁</td>
<td>1,1</td>
</tr>
<tr>
<td>t₂</td>
<td>0,0</td>
</tr>
<tr>
<td>t₃</td>
<td>0,0</td>
</tr>
</tbody>
</table>

- One “right” action for each type
- Coordination game
- Signals costless
Simple example

<table>
<thead>
<tr>
<th>Sender type</th>
<th>Receiver action</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td></td>
</tr>
<tr>
<td>$t_1$</td>
<td>1, 1</td>
<td>0, 0</td>
<td>0, 0</td>
<td></td>
</tr>
<tr>
<td>$t_2$</td>
<td>0, 0</td>
<td>1, 1</td>
<td>0, 0</td>
<td></td>
</tr>
<tr>
<td>$t_3$</td>
<td>0, 0</td>
<td>0, 0</td>
<td>1, 1</td>
<td></td>
</tr>
</tbody>
</table>

- One “right” action for each type
- Coordination game
- Signals costless
- Types of equilibria:
  - Separating (“signaling system”)

$t_1 \rightarrow s_1 \rightarrow a_1$
$t_2 \rightarrow s_2 \rightarrow a_2$
$t_3 \rightarrow s_3 \rightarrow a_3$
Simple example

Sender type

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>1, 1</td>
<td>0, 0</td>
<td>0, 0</td>
</tr>
<tr>
<td>$t_2$</td>
<td>0, 0</td>
<td>1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>$t_3$</td>
<td>0, 0</td>
<td>0, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

Receiver action

$t_1$ — $s_1$ — $a_1$
$t_2$ — $s_2$ — $a_2$
$t_3$ — $s_3$ — $a_3$

- One “right” action for each type
- Coordination game
- Signals costless
- Types of equilibria:
  - Separating ("signaling system")
  - Pooling
Simple example

<table>
<thead>
<tr>
<th>Sender type</th>
<th>Receiver action</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>$t_1$</td>
<td>1,1</td>
<td>0,0</td>
</tr>
<tr>
<td>$t_2$</td>
<td>0,0</td>
<td>1,1</td>
</tr>
<tr>
<td>$t_3$</td>
<td>0,0</td>
<td>0,0</td>
</tr>
</tbody>
</table>

- One “right” action for each type
- Coordination game
- Signals costless
- Types of equilibria:
  - Separating (“signaling system”)
  - Pooling
  - Partial Pooling
Information

- View the **information** of a signal as how it changes probabilities
- Signals involve two kinds of information:
  - What state the sender has observed
  - What action the receiver will take
- Information content and quantity
- Information is maximal in signaling system (but also in perfectly miscoordinating systems)
Information quantity

- Intuition: should compare probability with vs without observation
- Information quantity of signal $s$ "in favor of" state (type) $t$:
  \[ \log \frac{\sigma(t|s)}{\tau(t)} \]
- Overall information quantity of signal $s$:
  \[ \sum_{t \in T} \sigma(t|s) \log \frac{\sigma(t|s)}{\tau(t)} \]
  (Kullback-Leibler divergence)
- Information quantity of signal about act is analogous
Consider two equiprobable states $t_1, t_2$ and two signals $s_1, s_2$

Consider separating sender strategy $\sigma(t_1) = s_1, \sigma(t_2) = s_2$

Information quantity of $s_1$:

$$\sigma(t_1|s_1) \log \frac{\sigma(t_1|s_1)}{\tau(t_1)} + \sigma(t_2|s_1) \log \frac{\sigma(t_2|s_1)}{\tau(t_2)}$$

$$= 1 \log \frac{1}{0.5} + 0 \log \frac{1}{0.5} = 1 \text{ (bit)}$$

Consider pooling sender strategy $\sigma(t_1) = s_1, \sigma(t_2) = s_1$

Information quantity of $s_1$:

$$\sigma(t_1|s_1) \log \frac{\sigma(t_1|s_1)}{\tau(t_1)} + \sigma(t_2|s_1) \log \frac{\sigma(t_2|s_1)}{\tau(t_2)}$$

$$= \tau(t_1) \log \frac{\tau(t_1)}{\tau(t_1)} + \tau(t_1) \log \frac{\tau(t_1)}{\tau(t_1)} = 0 \text{ (bit)}$$
Information content

- “Meaning” of signal $s$
- Its information quantity in favor of each respective state

$$\langle \log \frac{\sigma(t_1|s)}{\tau(t_1)}, \ldots, \log \frac{\sigma(t_n|s)}{\tau(t_n)} \rangle$$

- Consider two equiprobable states $t_1, t_2$ and two signals $s_1, s_2$
- Consider separating sender strategy $\sigma(t_1) = s_1, \sigma(t_2) = s_2$
- Information content of $s_1$:

$$\langle 1, -\infty \rangle$$
Evolution

- **Replicator dynamics** as simple model of evolution
- Differential replication according to Darwinian fitness
- Discrete version proceeds in generations
- Equation to determine new proportion of individuals with strategy $s$:
  \[ x_{t+1}(s) = x_t(s) \frac{\text{Fitness}(s)}{\text{Average fitness}} \]
- Continuous version:
  \[ \dot{x}(s) = x \cdot (\text{Fitness}(s) - \text{Average fitness}) \]
- Fitness in the simplest case is payoff of random pairing
- For cooperation to evolve, correlation is needed
- For symmetry breaking and exploration, add random mutation
Depiction of replicator dynamics

- Unstable states, rest points, stable and strongly stable states
- Illustrating with Hawk-Dove, Prisoner’s dilemma, Inconsequential actions
Rock, scissors, paper

- Each pure strategy is equilibrium, but unstable
- Completely mixed state is stable, but not strongly
- No population that is not already in equilibrium converges
Evolution in signaling games

- Simplest case: two equiprobable types, two signals, two acts
- Sender and receiver have 4 strategies each, or 16 combined
- Signaling system always evolves
- All pooling equilibria are unstable
- Randomness breaks symmetry and creates information
Evolution in signaling games

- Simplest case: two equiprobable types, two signals, two acts
- Sender and receiver have 4 strategies each, or 16 combined
- Signaling system always evolves
- All pooling equilibria are unstable
- Randomness breaks symmetry and creates information
- With unequal probability, (partial) pooling equilibria may evolve
- The greater the inequality, the more likely
- On the other hand, the smaller the impact on the welfare is
Evolution in signaling games

- Simplest case: two equiprobable types, two signals, two acts
- Sender and receiver have 4 strategies each, or 16 combined
- Signaling system always evolves
- All pooling equilibria are unstable
- Randomness breaks symmetry and creates information

- With unequal probability, (partial) pooling equilibria may evolve
- The greater the inequality, the more likely
- On the other hand, the smaller the impact on the welfare is

- Details depend on the exact payoffs, probabilities and mutation rates
- Correlation can destabilize pooling
Deception

- Deception is ubiquitous in nature (e.g. Photuris vs Photinus)
- How can we define it, and how can it be sustainable?

- Deception is only meaningful in the context of an existing signaling convention
- Take the information content of a signal to be its agreed-upon meaning
- A signal whose information content does not reflect the type is misinformation (e.g., alarm call when no predator present)
- A misinformative signal benefitting the sender (and harming the receiver) is deceptive (e.g., Photuris)

- Systematic deception changes the convention (again, Photuris)
Successful deception in equilibrium

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>2,10</td>
<td>0,0</td>
<td>10,8</td>
</tr>
<tr>
<td>$t_2$</td>
<td>0,0</td>
<td>2,10</td>
<td>10,8</td>
</tr>
<tr>
<td>$t_3$</td>
<td>0,0</td>
<td>10,10</td>
<td>0,0</td>
</tr>
</tbody>
</table>

- Sender always manipulates receiver with “half-truths”:
  - In $t_1$, the sender’s signal raises the probability of $t_2$
  - In $t_2$, the sender’s signal raises the probability of $t_1$

- These half-truths induce receiver to choose $a_3$ in $t_1$ and $t_2$

- Sender benefits at expense of receiver (who prefers $a_1$ or $a_2$)

- Deception can even be seen as “morally good”:
  - Sender gains 8, receiver loses only 2
  - If you don’t know your role in advance (or you alternate), you would choose the deceptive equilibrium as universal law
Information bottlenecks can impact efficiency

![Table]

- Both are evolutionarily stable, although the right one is worse
Inventing new signals

- **Chinese restaurant process:**
  - Restaurant with infinite number of tables
  - Guests enter one by one
  - If $N$ guests are there, each new guest joins the table of any of them with probability $\frac{1}{N+1}$
  - With probability $\frac{1}{N+1}$, he starts a new table

- **Pólya urn process:**
  - Urn with various colored balls
  - Draw a ball at random, put back two of that color
  - “Neutral” evolution (without selection pressure)
  - Converges to random color

- **Hoppe-Pólya urn:**
  - Add a black “mutator” ball to Pólya’s urn
  - If it is drawn, put it back and add one with a new color
  - Equivalent to Chinese restaurant
  - Model for neutral evolution with invention
Inventing new signals

- Use a Hoppe-Pólya urn to model sender strategy
- Reinforcement learning: add balls depending on communication success (payoff)
- If receiver receives a new signal, he acts at random
- On success, the new signal is reinforced, otherwise removed
- Noisy forgetting to keep number of signals from exploding: at each step remove some ball at random
- In experiments, efficient signaling evolves quite robustly
Further topics

- Logic and information processing
- Complex signals and compositionality
- Teamwork
  - Quorum sensing (e.g. Vibrio fisheri)
  - Myxococcus xanthus
  - Multicellular organisms
- Learning to network
- Cheap talk