PSL Model Checking and Run-time Verification via Testers Technical Report: TR2006-881

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Abstract. The paper introduces the construct of *temporal testers* as a compositional basis for the construction of automata corresponding to temporal formulas in the PSL logic. Temporal testers can be viewed as (non-deterministic) transducers that, at any point, output a boolean value which is 1 iff the corresponding temporal formula holds starting at the current position.
The main advantage of testers, compared to acceptors (such as Büchi automata) is that they are compositional. Namely, a tester for a compound formula can be constructed out of the testers for its sub-formulas. In this paper, we extend the application of the testers method from LTL to the logic PSL.
Besides providing the construction of testers for PSL, we indicate how the symbolic representation of the testers can be directly utilized for efficient model checking and run-time monitoring.

1 Introduction

The standard way of model checking an LTL property φ over a finite-state system S, represented by the automaton M_S, is based on the construction of an ω-automaton $\mathcal{A}_{\neg\varphi}$ that accepts all sequences that violate the property φ. Having both the system and its specification represented by automata, we may form the product automaton $M_S \times \mathcal{A}_{\neg\varphi}$ and check that it accepts the empty language, implying that there exists no computation of S which refutes φ [14].

Usually, the automaton $\mathcal{A}_{\neg\varphi}$ is a non-deterministic Büchi automaton, which is constructed using an explicit-state representation. In order to employ it in a symbolic (BDD-based) model checker, it is necessary to encode the automaton by the introduction of auxiliary variables. Another drawback of the normal (tableau-based) construction is that it is not compositional. That is, having constructed automata $\mathcal{A}_\varphi$ and $\mathcal{A}_\psi$ for LTL formulas φ and ψ, there is no simple recipe for constructing the automaton for a compound formula which combines φ and ψ, such as $\varphi\, U\, \psi$.

The article [10] introduces a compositional approach to the construction of automata corresponding to LTL formulas. This construction is based on the notion of a *temporal tester* that has been introduced first in [9]. A tester for an LTL formula φ can be viewed as a *transducer* that keeps observing a state sequence σ and, at every position $j \geq 0$, outputs a boolean value which equals 1 iff $(\sigma, j) \models \varphi$. While acceptors, such as the Büchi automaton $\mathcal{A}_\varphi$, do not compose, transducers do. In Fig. 1, we show how transducers for the formulas φ, ψ, and $p\, U\, q$ can be composed into a transducer for the formula $\varphi\, U\, \psi$.
There are several important advantages to the use of temporal testers as the basis for the construction of automata for temporal formulas:

- The construction is compositional. Therefore, it is sufficient to specify testers for the basic temporal formulas: $X!p$ and $p\, U\, q$, where p and q are assertions (state formulas). Testers for more complex formulas can be derived by composition as in Fig. 1 .
- The testers for the basic formulas are naturally symbolic. Thus, a general tester, which is a synchronous parallel composition (automata product) of symbolic modules can also be easily represented symbolically.

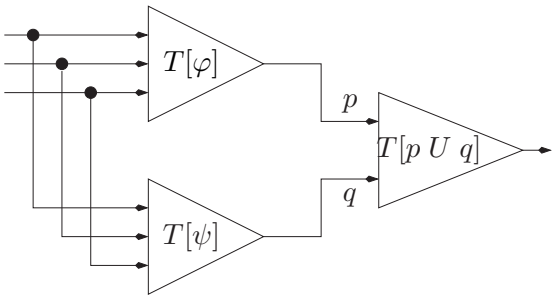

Fig. 1. Composition of transducers to form $T[\varphi\ U\ \psi]$.

- As shown below, the basic processes of model checking and run-time monitoring can be performed directly on the symbolic representation of the testers. There is no need for determinization or reduction to explicit state representation.

In spite of these advantages, the complexity of constructing a transducer (temporal tester) for an arbitrary LTL formula is not worse than that of the lower-functionality acceptor. In its symbolic representation, the size of a tester is linear in the size of the formula. This implies that the worst-case state complexity is exponential.

In the work presented in this paper, we generalize the temporal tester approach to the more expressive logic PSL, recently introduced as a new standard logic for specifying hardware properties [1]. Due to compositionality, it is only necessary to provide the construction of testers for the basic operators introduced by PSL.

In addition, we show how to construct an optimal symbolic run-time monitor. By optimality, we mean that the monitor extracts as much information as possible from the observed trace. In particular, an optimal monitor stops as soon it can be deduced that the specification is violated or satisfied, regardless of the possible continuations of the observed trace.

2 Accellera PSL

In this section, we are going to follow [6] and formally define the logic PSL. However, we only consider a subset of PSL. For brevity, we omit the discussions of OBE (Optional Branching Extension) formulas that are based on CTL. Note that using testers we can obtain a model checking algorithm even for CTL* branching formulas by combining PSL testers with the work in [10]. Regarding run-time monitoring, which together with model checking is the primary motivation for our work, branching formulas are not applicable at all. In addition, we only consider unclocked formulas. This is not a severe limitation since clocks do not add any expressive power to PSL [6].

2.1 Syntax

The logic Accellera PSL is defined with respect to a non-empty set of atomic propositions P. Let B be the set of boolean expressions over P. We assume that the expressions *true* and *false* belong to B.

Definition 1 (Sequential Extended Regular Expressions (SEREs)) .

– *Every boolean expression $b \in B$ is a SERE.*

– *If* r, r_1, *and* r_2 *are SEREs, then the following are SEREs:*
  - $\{r\}$
  - $r_1 ; r_2$
  - $r_1 : r_2$
  - $r_1 \mid r_2$
  - $[*0]$
  - $r_1 \,\&\&\, r_2$
  - $r[*]$

Definition 2 (Formulas of the Foundation Language (FL formulas)) .

– *If* r *is a SERE, then both* r *and* $r!$ *are FL formulas.*
– *If* φ *and* ψ *are FL formulas,* r *is a SERE, and* b *is a boolean expression, then the following are FL formulas:*
  - (φ)
  - $\neg\varphi$
  - $\varphi \wedge \psi$
  - $\langle r \rangle \varphi$
  - $X!\varphi$
  - $[\varphi \; U \; \psi]$
  - φ *abort* b
  - $r \mapsto \varphi$

Definition 3 (Accellera PSL Formulas) .

– *Every FL formula is an Accellera* PSL *formula.*

2.2 Semantics

The semantics of FL is defined with respect to finite and infinite words over $\Sigma = 2^P \cup \{\top, \bot\}$. We denote a letter from Σ by l and an empty, finite, or infinite word from Σ by u, v, or w (possibly with subscripts). We denote the length of word v as $|\, v \,|$. An empty word $v = \epsilon$ has length 0, a finite word $v = (l_0 l_1 l_2 \ldots l_k)$ has length $k+1$, and an infinite word has length ω. We use i, j, and k to denote non-negative integers. We denote the i^{th} letter of v by v^{i-1} (since counting of letters starts at zero). We denote by $v^{i..}$ the suffix of v starting at v^i. That is, for every $i < |v|$, $v^{i..} = v^i v^{i+1} \cdots v^n$ or $v^{i..} = v^i v^{i+1} \cdots$. We denote by $v^{i..j}$ the finite sequence of letters starting from v^i and ending in v^j . That is, for $j \geq i$, $v^{i..j} = v^i v^{i+1} \cdots v^j$ and for $j < i, v^{i..j} = \epsilon$. We use l^ω to denote an infinite-length word, each letter of which is l.

We use $\bar{v}$ to denote the word obtained by replacing every $\top$ with a $\bot$ and vice versa. We call $\bar{v}$ the *complement* of v.

The semantics of FL *formulas* over *words* is defined inductively, using as the base case the semantics of *boolean expressions* over *letters* in Σ. The semantics of a boolean expression is assumed to be given as a relation $\|\!= \subseteq \Sigma \times B$ relating letters in Σ with boolean expressions in B. If $(l, b) \in \|\!=$, we say that the letter l satisfies the boolean expression b and denote it by $l \|\!= b$. We assume the two special letters $\top$ and $\bot$ behave as follows: for every boolean expression b, $\top \|\!= b$ and $\bot \|\!\not= b$. We assume that, otherwise, the boolean relation $\|\!=$ behaves in the usual manner. In particular, that for every letter $l \in 2^P$, atomic proposition $p \in P$ and boolean expressions $b, b_1, b_2 \in B$, (i) $l \|\!= p$ iff $p \in l$, (ii) $l \|\!= \neg b$ iff $l \|\!\not= b$, and (iii) $l \|\!= \mathit{true}$ and $l \|\!\not= \mathit{false}$. Finally, we assume that for every letter $l \in \Sigma$, $l \|\!= b_1 \wedge b_2$ iff $l \|\!= b_1$ and $l \|\!= b_2$.

Semantics of SEREs SEREs are defined over finite words from the alphabet Σ. The notation $v \models r$, where r is a SERE and v a finite word means that v *tightly models* r. The semantics of unclocked SEREs are defined as follows, where b denotes a boolean expression, and r, r_1, and r_2 denote unclocked SEREs.

– $v \models \{r\} \iff v \models r$
– $v \models b \iff |v| = 1 \wedge v^0 \|\!= b$
– $v \models r_1 ; r_2 \iff \exists v_1, v_2$ s.t. $v = v_1 v_2, v_1 \models r_1$ and $v_2 \models r_2$
– $v \models r_1 : r_2 \iff \exists v_1, v_2,$ and l s.t. $v = v_1 l v_2, v_1 l \models r_1$ and $l v_2 \models r_2$
– $v \models r_1 \mid r_2 \iff v \models r_1$ or $v \models r_2$
– $v \models r_1 \,\&\&\, r_2 \iff v \models r_1$ and $v \models r_2$
– $v \models [*0] \iff v = \epsilon$
– $v \models r[*] \iff v = \epsilon$ or $\exists v_1, v_2$ s.t. $v_1 \neq \epsilon, v = v_1 v_2$ and $v_1 \models r$ and $v_2 \models r[*]$

Semantics of FL Let v be a finite or infinite word, b be a boolean expression, r be a SERE, and φ, ψ be FL formulas. We use $\models$ to define the semantics of FL formulas. If $v \models \varphi$ we say that v models (or satisfies) φ.

- $v \models (\varphi) \iff v \models \varphi$
- $v \models \neg\varphi \iff \bar{v} \not\models \varphi$
- $v \models \varphi \wedge \psi \iff v \models \varphi$ and $v \models \psi$
- $v \models b! \iff |v| > 0$ and $v^0 \Vdash b$
- $v \models b \iff |v| = 0$ or $v^0 \Vdash b$
- $v \models r! \iff \exists j < |v|$ s.t. $v^{0..j} \Vdash r$
- $v \models r \iff \forall j < |v|,\ v^{0..j} \top^\omega \models r!$
- $v \models X!\varphi \iff |v| > 1$ and $v^{1..} \models \varphi$
- $v \models [\varphi\ U\ \psi] \iff \exists k < |v|$ s.t. $v^{k..} \models \psi$, and $\forall j < k,\ v^{j..} \models \varphi$
- $v \models \varphi\ \mathit{abort}\ b \iff v \models \varphi$ or $\exists j < |v|$ s.t. $v^j \Vdash b$ and $v^{0..j-1} \top^\omega \models \varphi$
- $v \models \langle r \rangle \varphi \iff \exists j < |v|$ s.t. $\bar{v}^{0..j} \Vdash r,\ v^{j..} \models \varphi$
- $v \models r \mapsto \varphi \iff \forall j < |v|$ s.t. $\bar{v}^{0..j} \Vdash r,\ v^{j..} \models \varphi$

3 Computational Model

3.1 Fair Discrete Systems with Finite Computations

We take a *just discrete system*(JDS), which is a variant of *fair transition system* [11], as our computational model. Under this model, a system $\mathcal{D} : \langle V, \Theta, R, \mathcal{J}, F \rangle$ consists of the following components:

- V: A finite set of *system variables*. A *state* of the system $\mathcal{D}$ provides a type-consistent interpretation of the system variables V. For a state s and a system variable $v \in V$, we denote the value assigned to v by the state s by $s[v]$.
- Θ: The *initial condition*. This is an assertion (state formula) characterizing the initial states. A state is defined to be *initial* if it satisfies Θ.
- $R(V, V')$: The *transition relation*, which is an assertion that relates the values of the variables in V interpreted by a state s to the values of the variables V' in an R-*successor* state s'.
- $\mathcal{J}$: A set of *justice* (*weak fairness*) requirements. Each justice requirement is an assertion. An infinite computation must include infinitely many states satisfying the assertion.
- F: The *termination condition*, which is an assertion specifying the set of *final* states. Each finite computation must end in a final state.

A *computation* of an JDS $\mathcal{D}$ is a non-empty sequence of states $\sigma : s_0, s_1, s_2, ...$, satisfying the requirements:

- *Initiality*: s_0 is initial.
- *Consecution*: For each $i \in [0, |\sigma|)$, the state s_{i+1} is a R-successor of state s_i. That is, $\langle s_i, s_{i+1} \rangle \in R(V, V')$ where, for each $v \in V$, we interpret v as $s_i[v]$ and v' as $s_{i+1}[v]$.
- *Justice*: If σ is infinite, then for every $J \in \mathcal{J}$, σ contains infinitely many occurrences of J-states.
- *Termination*: If $\sigma = s_0, s_1, s_2, ..., s_k$ (i.e., σ is finite), then s_k must satisfy F.

A sequence of states $\sigma : s_0, s_1, s_2, ...$ that only satisfies all conditions for being a computation except initiality is called an *uninitialized computation*. A sequence of states $\sigma : s_0, s_1, s_2, ...$ that only satisfies consecution is called an *uninitialized run*.

Given two JDS's, $\mathcal{D}_1$ and $\mathcal{D}_2$, their *synchronous parallel composition*, $\mathcal{D}_1 \mid\mid\mid \mathcal{D}_2$, is the JDS whose sets of variables and justice requirements are the unions of the corresponding sets in the two systems, whose initial and termination conditions are the conjunctions of the corresponding assertions, and whose transition relation is defined as the conjunction of the two transition relations. Thus, a step in an execution of the composed system is a joint step of the systems $\mathcal{D}_1$ and $\mathcal{D}_2$.

3.2 Interpretation of PSL formulas over JDS

We assume that the set of atomic propositions P is a subset of the variables V, so we can easily evaluate all the propositions at a given state of a JDS. We say that a letter $l \in 2^P$ *corresponds* to a state s if $p \in l$ iff $s[p] = 1$. Similarly, we define a correspondence between words and computations. We say, that a computation σ models (or satisfies) PSL formula φ, denoted $\sigma \vDash \varphi$, if the corresponding word v satisfies PSL formula φ.

4 Temporal Testers

One of the main problems in constructing a Büchi automaton for a PSL formula (or for that matter any ω-regular language) is that the conventional construction is not compositional. In particular, given Büchi automata $\mathcal{A}_\varphi$ and $\mathcal{A}_\psi$ for formulas φ and ψ, it is not trivial to build an automaton for $\varphi \, U \, \psi$. Compositionality is an important consideration, especially in the context of PSL. It is expected that specifications are written in a modular way, and PSL has several language constructs to facilitate that. For example, any property can be given a name, and a more complex property can be built by simply using a named sub-property instead of an atomic proposition.

One way to achieve compositionality with Büchi automata is to use alternation [3]. Nothing special is required from the Büchi automata to be composed in such manner, but the presence of universal branching in the resulting automaton is undesirable. Though most model checkers can deal with existential non-determinism directly and efficiently, universal branching is usually preprocessed at exponential cost.

Our approach is based on the observation that while there is very little room to maneuver during the merging step of two Büchi automata, the construction process of the sub-components is wide open for a change. In particular, we suggest that each sub-component assumes the responsibility of being easily composed with other parts. The hope is that, by requiring individual parts be more structured than the traditional Büchi automata, we can significantly simplify the composition process.

Recall that the main property of Büchi automata (as well as any other automata) is to correctly identify a language membership of a given sequence of letters, starting from the very first letter. It turns out that for composition it is also very useful to know whether a word is in the language starting from an arbitrary position i. We refer to this new class of objects as *testers*. Essentially, testers are transducers that at each step output whether the suffix of the input sequence is in the language. Of course, the suffix is not known by the time the decision has to be made, so the testers are inherently non-deterministic.

Formally, a *tester* for a PSL formula φ is a JDS T_φ, which has a distinguished boolean variable x_φ, such that:

- **Soundness:** For every computation $\sigma : s_0, s_1, s_2, ...$ of T_φ , $s_i[x_\varphi] = 1$ iff $\sigma^{i..} \models \varphi$
- **Completeness:** For every sequence of states $\sigma' : s'_0, s'_1, s'_2, ...$, there is a matching computation $\sigma : s_0, s_1, s_2, ...$ such that for each i, s_i and s'_i agree on the interpretation of φ-variables.

Intuitively, the second condition requires that a tester must be able to correctly interpret x_φ for an arbitrary input sequence. Otherwise, the first condition can be trivially satisfied by a JDS that has no computations.

5 LTL Testers

We are going to continue the presentation of testers by considering two very important PSL operators, namely $X!$(next) and U(until). First, we show how to build testers for two *basic formulas* $X!b$ and $b_1\ U\ b_2$, where b, b_1, and b_2 are boolean expressions. Then, we demonstrate high compositionality of the testers by easily extending the result to cover full LTL. Note that our construction for LTL operators is very similar to the one presented in [9].

5.1 A Tester for $\varphi = X!b$

Let $T_\varphi = \langle V_\varphi, \Theta_\varphi, R_\varphi, \mathcal{J}_\varphi, F_\varphi \rangle$ be the tester we wish to construct. The components of T_φ are defined as follows:

$$T(X!b): \begin{cases} V_\varphi : P \cup x_\varphi, \text{ where } P \text{ is a set of atomic propositions used to construct } B \\ \Theta_\varphi : 1 \\ R_\varphi(V, V') : x_\varphi = b' \\ \mathcal{J}_\varphi : \emptyset \\ F_\varphi : \neg x_\varphi \end{cases}$$

It almost immediately follows from the construction that $T(X!b)$ is indeed a good tester for $X!b$. The soundness of the $T(X!b)$ is guaranteed by the transition relation with the exception that we still have a freedom to incorrectly interpret x_φ at the very last state. This case is handled separately by insisting that every final state must interpret x_φ as *false*. The completeness follows from the fact that we do not restrict P variables, in any way, by the transition relation, and we can always interpret x_φ properly, by either matching b' or setting it to *false* in the last state.

5.2 A Tester for $\varphi = b_1\ U\ b_2$

The components of T_φ are defined as follows:

$$T(b_1\ U\ b_2): \begin{cases} V_\varphi : P \cup x_\varphi \\ \Theta_\varphi : 1 \\ R_\varphi(V, V') : x_\varphi = b_2 \vee (b_1 \wedge x'_\varphi) \\ \mathcal{J}_\varphi : b_2 \vee \neg x_\varphi \\ F_\varphi : b_2 \vee \neg x_\varphi \end{cases}$$

Unlike the previous tester, $T(b_1\ U\ b_2)$ has a non-empty justice set. A technical reason is that the transition relation allows x_φ to be continuously set to true without having a single state that actually satisfies b_2. The situation is ruled out by the justice requirement. Another way to look at the problem is that R_φ represents an expansion formula for the U(strong until) operator, namely $b_1\ U\ b_2 \iff b_2 \vee (b_1 \wedge X![b_1\ U\ b_2])$. In general, starting with an expansion formula is a good first step when building a tester. However, the expansion formula alone is usually not sufficient for a proper tester. Indeed, consider the operator $\mathcal{W}$(weak until), defined as $b_1\ \mathcal{W}\ b_2 \equiv \neg(true\ U\ \neg b_1) \vee b_1\ U\ b_2$, which has exactly the same expansion formula, namely $b_1\ \mathcal{W}\ b_2 \iff b_2 \vee (b_1 \wedge X![b_1\ \mathcal{W}\ b_2])$. We use justice to differentiate between the two operators.

6 Tester Composition

In Fig. 2, we present a recursive algorithm that builds a tester for an arbitrary LTL formula φ. In Example 1, we illustrate the algorithm by applying the tester construction for the formula $\varphi = \mathit{true}\ U\ \big(X![b_1\ U\ b_2] \vee (b_3\ U\ [b_1\ U\ b_2])\big)$.

- **Base Case**: If φ is a basic formula (i.e., $\varphi = X!b$ or $\varphi = b_1\ U\ b_2$), use construction from Section 5. For a trivial case, when the formula φ does not contain any temporal operators, we can use a tester for $\mathit{false}\ U\ \varphi$.
- **Induction Step**: Let ψ be an innermost basic sub-formula of φ, then $T_\varphi = T_{\varphi[\psi/x_\psi]}\ |||\ T_\psi$, where $\varphi[\psi/x_\psi]$ denotes the formula φ in which each occurrence of the sub-formula ψ is replaced with x_ψ.

Fig. 2. Tester construction for an arbitrary LTL formula φ

Example 1 *Tester Construction for* $\varphi = \mathit{true}\ U\ \big(X![b_1\ U\ b_2] \vee (b_3\ U\ [b_1\ U\ b_2])\big)$

We start by identifying $b_1\ U\ b_2$ to be the innermost basic sub-formula and building the corresponding tester, $T_{b_1 U b_2}$. Assume that z is the output variable of the tester $T_{b_1 U b_2}$. Let $\alpha = \varphi[b_1\ U\ b_2/z]$; after the substitution $\alpha = \mathit{true}\ U\ \big(X!z \vee (b_3\ U\ z)\big)$. Note that we performed the substitution twice, but there is no need for two testers, which can result in significant savings. We proceed in similar fashion and build two more testers $T_{X!z}$ and $T_{b_3 U z}$ with the output variables x and y. After the substitutions, we obtain $\beta = \mathit{true}\ U\ [x \vee y]$. Since $x \vee y$ is just a boolean expression, the formula satisfies the condition of the base case, and we can finish the construction with one more step. The final result can be expressed as:

$$T_\varphi = T_\beta\ |||\ T_{X!z}\ |||\ T_{b_3 U z}\ |||\ T_{b_1 U b_2}.$$

Though we have assumed φ is an LTL formula, the algorithm is applicable for PSL as well. The only extension necessary is the ability to deal with additional basic formulas.

7 Associating a Regular Grammar with a SERE

Following [8], a grammar $\mathcal{G} = \langle \mathcal{V}, \mathcal{T}, \mathcal{P}, \mathcal{S} \rangle$ consists of the following components:

- $\mathcal{V}$: A finite set of *variables*.
- $\mathcal{T}$: A finite set of *terminals*. We assume that $\mathcal{V}$ and $\mathcal{T}$ are disjoint. In our framework, $\mathcal{T}$ consists of boolean expressions and a special terminal ϵ.
- $\mathcal{P}$: A finite set of *productions*. We only consider right-linear grammars, so all productions are of the form $V \rightarrow aW$ or $V \rightarrow a$, where a is a terminal, and V and W are variables.
- $\mathcal{S}$: A special variable called a *start symbol*.

We say a grammar $\mathcal{G}$ is *associated* with a SERE r if, intuitively, they both define the same language. While this definition is not accurate, we show a precise construction of an associated grammar for a given SERE in Appendix A. For example, we associate the following grammar $\mathcal{G}$ with SERE $r = (a_1 b_1)[*]\ \&\&\ (a_2 b_2)[*]$

$$V_1 \rightarrow \epsilon \mid (a_1 \wedge a_2) V_2$$
$$V_2 \rightarrow \quad (b_1 \wedge b_2) V_1$$

Theorem 1. *For every SERE r of length n, there exists an associated grammar $\mathcal{G}$ with the number of productions $O(2^n)$. If we restrict SERE's to the three traditional operators: concatenation (;), union (|), and Kleene closure ([*]), the number of productions becomes linear in the size of r.*

8 PSL Testers

As we have mentioned before, to handle the full PSL it is enough to handle all the basic PSL formulas. More complicated formulas can be handled via tester composition according to the algorithm in Fig. 2. There are only two additional PSL basic formulas that we need to consider, namely $\varphi = \langle r \rangle b$ and $\varphi = r$, where r is a SERE and b is a boolean expression. All other PSL temporal operators can be expressed using those two and the LTL operators, $X!$ and U. For example, $r! \equiv \langle r \rangle \mathit{true}$, and $r \mapsto b \equiv \neg(\langle r \rangle \neg\varphi)$. The *abort* operator is a little bit more complicated, and we present a set of rewriting rules in Section 8.3.

8.1 A Tester for $\varphi = \langle r \rangle b$

Let $T_\varphi = \langle V_\varphi, \Theta_\varphi, R_\varphi, \mathcal{J}_\varphi, F_\varphi \rangle$ be the tester we wish to construct. Assume that x_φ is the output variable. Let $\mathcal{G} = \langle \mathcal{V}, \mathcal{T}, \mathcal{P}, \mathcal{S} \rangle$ be a grammar associated with r. Without the loss of generality, we assume $\mathcal{G}$ has variables $V_1, \ldots, V_n$ with V_1 being the start symbol. In addition, each variable V_i, has derivations of the form:

$$V_i \rightarrow \alpha_1 \mid \cdots \mid \alpha_m \mid \beta_1 V_1 \mid \cdots \mid \beta_n V_n$$

where $\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_n$ are boolean expressions. The case that variable V_i does not have a particular derivation $V_i \rightarrow \beta_j V_j$ or $V_i \rightarrow \alpha_k$, is covered by having $\beta_j = \mathit{false}$, and similarly, $\alpha_k = \mathit{false}$. Note that by insisting on this specific form, which does not allow ϵ productions, we can not express whether an empty string is in the language. However, since, by definition of $\langle\rangle$ operator, a prefix that satisfies r must be non-empty, we do not need to consider this. The tester T_φ is given by:

$$T(\langle r \rangle b): \begin{cases} V_\varphi : P \cup x_\varphi \cup \{X_1, \ldots, X_n, Y_1, \ldots, Y_n\} \\ \Theta_\varphi : 1 \\ R_\varphi(V, V') : \text{Each derivation } V_i \rightarrow \alpha_1 \mid \cdots \mid \alpha_m \mid \beta_1 V_1 \mid \cdots \mid \beta_n V_n \\ \qquad \text{contributes to } \rho \text{ the conjunct} \\ \qquad X_i = (\alpha_1 \wedge b) \vee \cdots \vee (\alpha_m \wedge b) \vee (\beta_1 \wedge X_1') \vee \cdots \vee (\beta_n \wedge X_n') \\ \qquad \text{and the conjunct} \\ \qquad Y_i \rightarrow (\alpha_1 \wedge b) \vee \cdots \vee (\alpha_m \wedge b) \vee (\beta_1 \wedge Y_1') \vee \cdots \vee (\beta_n \wedge Y_n') \\ \qquad \text{the output variable is constrained by the conjunct} \\ \qquad x_\varphi = X_1 \\ \mathcal{J}_\varphi : \{\neg Y_1 \wedge \cdots \wedge \neg Y_n, \quad X_1 = Y_1 \wedge \cdots \wedge X_n = Y_n\} \\ F_\varphi : \text{Each derivation } V_i \rightarrow \alpha_1 \mid \cdots \mid \alpha_m \mid \beta_1 V_1 \mid \cdots \mid \beta_n V_n \\ \qquad \text{contributes to } F \text{ the conjunct} \\ \qquad X_i = (\alpha_1 \wedge b) \vee \cdots \vee (\alpha_m \wedge b) \end{cases}$$

Example 2 *A Tester for* $\varphi = \langle \{pq\}[*] \rangle b$.

To illustrate the construction, consider formula $\langle \{pq\}[*] \rangle b$. Following the algorithm from Appendix A and removing ϵ productions, the associated right-linear grammar for the SERE $\{pq\}[*]$ is given by

$V_1 \rightarrow \quad pV_2$
$V_2 \rightarrow q \mid qV_1$

Consequently, a tester for $\langle \{pq\}[*] \rangle b$ is given by

$$T(\langle\{pq\}[*]\rangle b): \begin{cases} V_\varphi : P \cup x_\varphi \cup \{X_1, X_2, Y_1, Y_2\} \\ \Theta_\varphi : 1 \\ R_\varphi(V, V') : \begin{pmatrix} (X_1 = (p \wedge X_2')) & \wedge \\ (X_2 = (q \wedge b) \vee (q \wedge X_1')) & \wedge \\ (Y_1 \rightarrow (p \wedge Y_2')) & \wedge \\ (Y_2 \rightarrow (q \wedge b) \vee (q \wedge Y_1')) & \wedge \\ x_\varphi = X_1 \end{pmatrix} \\ \mathcal{J}_\varphi : \{\neg Y_1 \wedge \neg Y_2, \quad X_1 = Y_1 \wedge X_2 = Y_2\} \\ F_\varphi : (X_1 = \mathit{false}) \wedge (X_2 = q \wedge b) \end{cases}$$

The variables $\{X_1, \ldots, X_n, Y_1, \ldots, Y_n\}$ are expected to check that the rest of the sequence from now on has a prefix satisfying the SERE r. Thus, the subsequence $s_j, \ldots, s_k, \ldots \vDash \langle r\rangle b$ iff there exists a generation sequence $V^j = V_1, V^{j+1}, \ldots, V^k$, such that for each $i, j \leq i < k$, there exists a grammar rule $V^i \rightarrow \beta V^{i+1}$, where $s_i \Vdash \beta, V^k \rightarrow \alpha,$ and $s_k \Vdash (\alpha \wedge b)$.

The generation sequence is represented in a run of the tester by a sequence of true valuations for the variables $Z^j = Z_1, Z^{j+1}, \ldots, Z^k$ where $Z^i \in \{X^i, Y^i\}$ for each $i \in [j..k]$. An important element in this checking is to make sure that any such generation sequence is finite. This is accomplished through the double representation of each V_i by X_i and Y_i. The justice requirement $(X_1 = Y_1) \wedge \cdots \wedge (X_n = Y_n)$ guarantees that that any true X_i is eventually copied into Y_i. The justice requirement $\neg Y_1 \wedge \cdots \wedge \neg Y_n$ guarantees that all true Y_i's are eventually falsified. Together, they guarantee that there exists no infinite generation sequence. The double representation approach was first introduced in [12].

8.2 A Tester for $\varphi = r$

We start the construction exactly the same way as we did for $\varphi = \langle r\rangle b$, in Section 8.1. Let $T_\varphi = \langle V_\varphi, \Theta_\varphi, R_\varphi, \mathcal{J}_\varphi, F_\varphi\rangle$ be the tester we wish to construct. Assume that x_φ is the output variable. Let $\mathcal{G} = \langle \mathcal{V}, \mathcal{T}, \mathcal{P}, \mathcal{S}\rangle$ be a grammar associated with r.

The tester T_φ is given by:

$$T(r): \begin{cases} V_\varphi : P \cup x_\varphi \cup \{X_1, \ldots, X_n, Y_1, \ldots, Y_n\} \\ \Theta_\varphi : 1 \\ R_\varphi(V, V') : \text{Each derivation } V_i \rightarrow \alpha_1 \mid \cdots \mid \alpha_m \mid \beta_1 V_1 \mid \cdots \mid \beta_n V_n \\ \qquad \text{contributes to } \rho \text{ the conjunct} \\ \qquad X_i = \alpha_1 \vee \cdots \vee \alpha_m \vee (\beta_1 \wedge X_1') \vee \cdots \vee (\beta_n \wedge X_n') \\ \qquad \text{and the conjunct} \\ \qquad \alpha_1 \vee \cdots \vee \alpha_m \vee (\beta_1 \wedge Y_1') \vee \cdots \vee (\beta_n \wedge Y_n') \rightarrow Y_i \\ \qquad \text{the output variable is constrained by the conjunct} \\ \qquad x_\varphi = X_1 \\ \mathcal{J}_\varphi : \{Y_1 \wedge \cdots \wedge Y_n, \quad X_1 = Y_1 \wedge \cdots \wedge X_n = Y_n\} \\ F_\varphi : \text{Each derivation } V_i \rightarrow \alpha_1 \mid \cdots \mid \alpha_m \mid \beta_1 V_1 \mid \cdots \mid \beta_n V_n \\ \qquad \text{contributes to } F \text{ the conjunct} \\ \qquad X_i = \alpha_1 \vee \cdots \vee \alpha_m \vee \beta_1 \vee \cdots \vee \beta_n \end{cases}$$

The variables $\{X_1, \ldots, X_n, Y_1, \ldots, Y_n\}$ are expected to check that the rest of the sequence from now on has a prefix that does not violate SERE r. We follow a similar approach as for the tester $\varphi = \langle r\rangle b$. However, now we are more concerned with false values of the variables $X_1 \ldots X_n$. The duality comes from the fact that, now, we are trying to prevent postponing the violation of the formula r forever.

8.3 Handling *abort* operator

To handle *abort*, we rewrite a given formula to a semantically equivalent one not containing the *abort* operator. Let φ be a given formula. Without loss of generality, assume that $\varphi = \psi$ *abort* b, where ψ is *abort*-free. An arbitrary formula can be processed by starting with an inner-most *abort* and removing them one by one.

For convenience, let *precedes* be the dual of *abort* such that ϕ *precedes* $b \equiv \neg(\neg\phi$ *abort* $b)$. Let f and g denote arbitrary PSL formulas; b, b_1, and b_2 denote boolean expressions; r denote a SERE. Let r_b be a SERE such that the formula $\phi = r_b$ is equivalent to $\phi = r$ *abort* b. A simple algorithm to construct r_b, given r, is presented at the end of this section. We use the following equivalencies to rewrite φ:

- b_1 *abort* $b_2 \equiv b_1 \vee b_2$
- b_1 *precedes* $b_2 \equiv b_1 \wedge \neg b_2$
- $(\neg f)$ *abort* $b \equiv \neg(f$ *precedes* $b)$
- $(\neg f)$ *precedes* $b \equiv \neg(f$ *abort* $b)$
- $(f \wedge g)$ *abort* $b \equiv (f$ *abort* $b) \wedge (g$ *abort* $b)$
- $(f \wedge g)$ *precedes* $b \equiv (f$ *precedes* $b) \wedge (g$ *precedes* $b)$
- $(X!f)$ *abort* $b \equiv b \vee X!(f$ *abort* $b)$
- $(X!f)$ *precedes* $b \equiv \neg b \wedge X!(f$ *precedes* $b)$
- $(f\ U\ g)$ *abort* $b \equiv (f$ *abort* $b)\ U\ (g$ *abort* $b)$
- $(f\ U\ g)$ *precedes* $b \equiv (f$ *precedes* $b)\ U\ (g$ *precedes* $b)$
- $(\langle r\rangle f)$ *abort* $b \equiv \langle r_b\rangle(f$ *abort* $b)$
- $(\langle r\rangle f)$ *precedes* $b \equiv \langle r$ &&$\neg b[*]\rangle(f$ *precedes* $b)$
- r *abort* $b \equiv r_b$
- r *precedes* $b \equiv r$ &&$\neg b[*]$

Note that the size of the resulting formula is linear in the size of the original. In addition, while && is usually a very expensive operator, it is benign in our case since a grammar for $\neg b[*]$ has only one non-terminal, and can be completely eliminated from the rewriting rules.

We conclude the discussion of the *abort* operator by elaborating on the construction of r_b. Let $\mathcal{G}$ be a grammar associated with r. Our goal is to construct $\mathcal{G}' = \langle \mathcal{V}', \mathcal{T}', \mathcal{P}', \mathcal{S}' \rangle$ - a grammar associated with r_b.

- $\mathcal{V}' = \mathcal{V} \cup \{V_f\}$
- $\mathcal{T}' = \mathcal{T} \cup \{b\}$
- $\mathcal{P}' = \mathcal{P} \cup \{V_f \rightarrow trueV_f, V_f \rightarrow \epsilon\} \cup \{V_i \rightarrow bV_f \mid V_i \in \mathcal{V}\}$
- $\mathcal{S}' = \mathcal{S}$

8.4 Complexity of the Construction

Theorem 2. *For every* PSL *formula φ of length n, there exists a tester with $O(2^n)$ variables. If we restrict SERE's to three traditional operators: concatenation* (;), *union* (|), *and Kleene closure* ([*]), *the number of variables is linear in the size of φ.*

To justify the result, we can just count the fresh variables introduced at each step of the tester construction. There is only linear number of sub-formulas, so there is a linear number of output variables. The only other variables introduced are the ones that are used to handle SERE's. According to Theorem 1, the associated grammars contain at most $O(2^n)$ non-terminals ($O(n)$ - for the restricted case). We conclude by observing that testers for the formulas $\varphi = \langle r\rangle b$ and $\varphi = r$ introduce exactly two variables, X_i and Y_i, for each non-terminal V_i.

9 Using Testers for Model Checking

One of the main advantages of our construction is that all the steps, as well as the final result – the tester itself, can be represented symbolically. That is particularly handy if one is to

use symbolic model checking [2]. Assume that the formula under consideration is φ, and $T_\varphi = \langle V_\varphi, \Theta_\varphi, R_\varphi, \mathcal{J}_\varphi, F_\varphi \rangle$ is the corresponding tester. Let JDS $\mathcal{D}$ represent the system we wish to model check.

We are going to use traditional automata theoretic approach based on synchronous composition, as in [2]. We perform the following steps:

- Compose $\mathcal{D}$ with T_φ to obtain $\mathcal{D} \,|||\, T_\varphi$.
- Check if $\mathcal{D} \,|||\, T_\varphi$ has a (fair) computation, such that $s_0[x_\varphi] = 0$. $\mathcal{D} \,|||\, T_\varphi$ has such a computation iff $\mathcal{D}$ does not satisfy φ.

As you can see, a tester can be used anywhere instead of an automaton. Indeed, we can always obtain an automaton from a tester by restricting the initial state to interpret x_φ as *true*.

10 Run-time Monitoring with Testers

The problem of *run-time monitoring* can be described as follows. Assume a reactive system $\mathcal{D}$ and a PSL formula φ, which formalizes a property that $\mathcal{D}$ should satisfy. In order to test the conjecture that $\mathcal{D}$ satisfies φ, we construct a program M, to which we refer as a *monitor*, that observes individual behaviors of $\mathcal{D}$. Behaviors of $\mathcal{D}$ are fed to the monitor state by state. After observing the finite sequence $\sigma : s_0, \ldots, s_k$ for some $k \geq 0$, we expect the monitor to be able to answer a subset of the following questions:

1. Does σ satisfy the formula φ?
2. Is φ *negatively determined* by σ? That is, is it the case that $\sigma \cdot \eta \not\models \varphi$ for all finite or infinite completions η.
3. Is φ *positively determined* by σ? That is, is it the case that $\sigma \cdot \eta \models \varphi$ for all finite or infinite completions η?
4. Is φ $\sigma-$*monitorable*? That is, is it the case that there exists a finite η such that φ is positively or negatively determined by $\sigma \cdot \eta$. If $\mathcal{D}$ is expected to run forever then it is useless to continue monitoring after observing σ such that φ is not $\sigma-$*monitorable*.

Solving the above questions leads to a creation of an *optimal* monitor - a monitor that extracts as much information as possible from the observation σ. In particular, an optimal monitor detects a violation of the property as early as possible. Of course, a monitor can do better if we supply it with some implementation details of the system $\mathcal{D}$, which may allow to deduce a violation even earlier [13]. In the extreme case, when a monitor knows everything about $\mathcal{D}$ the monitoring problem is reduced to model checking.

10.1 Monitoring with Testers

Let $\mathcal{D} : \langle P, \Theta, R, \mathcal{J}, F \rangle$ be a reactive system with observable variables P, and let φ be a PSL formula over P, which validity with respect to $\mathcal{D}$ we wish to test. Assume that $T_\varphi = \langle V_\varphi, \Theta_\varphi, R_\varphi, \mathcal{J}_\varphi, F_\varphi \rangle$ is the tester for φ, where the variables $V_\varphi = P \cup A$ are partitioned into the variables of $\mathcal{D}$ and additional auxiliary variables A. Let x_φ be the distinguished output variable of the tester T.

For an assertion (state formula) α, we define the R_φ-*predecessor* and R_φ-*successor* of α by

$$R_\varphi \diamond \alpha \;=\; \exists V'_\varphi : R_\varphi(V_\varphi, V'_\varphi) \wedge \alpha' \qquad \text{and} \qquad \alpha \diamond R_\varphi \;=\; \mathit{unprime}(\exists V_\varphi : R_\varphi(V_\varphi, V'_\varphi) \wedge \alpha),$$

where *unprime* simply replaces all next state variables with current state variable. Remember that the transition relation R_φ has two copies of each variable, one representing a current state and the other copy (a primed one) the next state.

Let $\sigma : s_0, s_1, \ldots, s_k$ be a finite observation produced by system $\mathcal{D}$. That is, a sequence of evaluations of the variables P. We define the *symbolic monitoring trace* $\mathcal{M} = \alpha_0, \alpha_1, \ldots, \alpha_k$ as the sequence of assertions given by

$$\alpha_0 = \Theta_\varphi \wedge x_\varphi \wedge (P{=}s_0), \text{ and } \alpha_{i+1} = (\alpha_i \diamond R_\varphi) \wedge (P{=}s_{i+1}) \text{ for all } i < k,$$

where $P = s$ stands for $\bigwedge_{v \in P} v = s[v]$.

Essentially, α_i represents a "current" state of the monitor, which is more precisely just a set of states of the tester T_φ. Whenever, the system makes a step from s_i to s_{i+1}, a monitor takes the corresponding step from α_i to α_{i+1} according to the transition relation R_φ and the interpretation of the propositions by the state s_{i+1}. The whole process can be described as, on the fly, synchronous, composition of the system and the tester, in which the later is determinized using classical subset construction. Note that we only need to worry about the existential non-determinism, A similar approach, but for alternating automata was also used for a so called breadth-first traversal in [7]. The monitoring sequence can be used to answer the first of the monitoring questions as stated by the following claim:

Claim 1 (Finitary satisfaction) *For a* PSL *formula φ, the finite sequence $\sigma : s_0, s_1, \ldots, s_k$ satisfies φ, i.e., $\sigma \vDash \varphi$, iff the formula $\alpha_k \ \wedge \ F_\varphi$ is satisfiable.*

The correctness of the claim results from the following observations. The tester T_φ can be interpreted as a non-deterministic automaton for acceptance of sequences satisfying φ if we insist that x_φ is *true* in the initial state. Furthermore, the assertion α_k represents all the automaton (tester) states which can be reached after reading the input σ. If any such evaluation is consistent with the assertion F_φ, which represents the set of final states, then this points to an accepting run of the automaton.

10.2 Deciding Negative Determinacy

Claim 1 has settled the first monitoring task. Next we consider one of the remaining tasks. Namely, we show how to decide whether, for a given σ, $\sigma \cdot \eta \not\vDash \varphi$ for all infinite or finite completions η.

In order to do this, we have to perform some offline calculations as a preparation. We generalize the notion of a single-step predecessor to an *eventual* predecessor by defining

$$R^*_\varphi \diamond \alpha = \alpha \vee R_\varphi \diamond \alpha \vee R_\varphi \diamond (R_\varphi \diamond \alpha) \vee \cdots$$

Consider the fix-point expression presented in Equation (1).

$$\mathit{feas} \quad = \quad [\mu X : (R_\varphi \diamond X) \vee F_\varphi] \quad \bigvee \quad [\nu Y : R_\varphi \diamond Y \ \wedge \ \bigwedge_{J \in \mathcal{J}} R^*_\varphi \diamond (Y \wedge J_\varphi)] \tag{1}$$

The first expression captures all the states that have a path to a final state. The second expression captures a maximal set of tester states Y such that every non-final state $s \in Y$ has an Y-successor and, for every justice requirement J, s has a Y-path leading to some Y-state which also satisfies J. The following can be proven:

Claim 2 (Feasible states) *The set feas characterizes the set of all states which originate an uninitialized computation.*

Assuming that we have precomputed the assertion *feas*, the following claim tells us how to decide whether a finite observation σ is sufficient in order to negatively determine φ:

Claim 3 (Negative Determinacy) *The* PSL *formula φ is negatively determined by the finite observation $\sigma = s_0, s_1, \ldots, s_k$ iff $\alpha_k \wedge feas$ is unsatisfiable.*

The claim is justified by the observation that $\alpha_k \wedge feas$ being unsatisfiable means that there is no way to complete the finite observation σ into a finite or infinite observation which will satisfy φ.

10.3 Deciding Positive Determinacy

In order to decide positive determinacy, we need to monitor the incoming observations not only by assertion sequences which attempt to validate φ but also by an assertion sequence which attempts to refute φ. Consequently, we define the *negative symbolic monitoring trace* $\mathcal{M}^- = \beta_0, \beta_1, \ldots, \beta_k$ by

$$\beta_0 = \Theta_\varphi \wedge \neg x_\varphi \wedge (P{=}s_0), \text{ and } \beta_{i+1} = (\beta_i \diamond R_\varphi) \wedge (P{=}s_{i+1}) \text{ for all } i < k$$

Claim 4 (Positive Determinacy) *The* PSL *formula φ is positively determined by the finite observation $\sigma = s_0, s_1, \ldots, s_k$ iff $\beta_k \wedge feas$ is unsatisfiable.*

10.4 Detecting Non-Monitorable Prefixes

Unfortunately, not all properties can be effectively monitored. Consider a property $\Box \diamondsuit p$, which is not σ-monitorable for any σ prefix. No useful information can be gained after observing a finite prefix if the property only depends on the things that must happen infinitely often. A good monitor should be able to detect such situations and alert the user. Next, we show how to decide whether φ is σ-monitorable, for a given σ.

Let $\mathcal{M} = \alpha_0, \alpha_1, \ldots, \alpha_k$ and $\mathcal{M}^- = \beta_0, \beta_1, \ldots, \beta_k$ be the positive and negative symbolic monitoring traces that correspond to σ. Let Γ represent a set of assertions. We define the *R_φ-successor* and *eventual R_φ-successor* of Γ by

$$\Gamma \diamondsuit R_\varphi \;=\; \{(\gamma \diamond R_\varphi) \wedge (P = s) \mid \gamma \in \Gamma, s \text{ is some state of the system } \mathcal{D}\}$$

and

$$\Gamma \diamondsuit R_\varphi^* = \Gamma \vee R_\varphi \diamondsuit \Gamma \vee R_\varphi \diamondsuit (R_\varphi \diamondsuit \Gamma) \vee \cdots$$

Claim 5 (Monitorability) *A* PSL *formula φ is $\sigma-$monitorable, where $\sigma = s_0, s_1, \ldots, s_k$, iff there exists an assertion γ such that either $\gamma \in (\alpha_k \diamondsuit R_\varphi^*)$ or $\gamma \in (\beta_k \diamondsuit R_\varphi^*)$, and $(\gamma \wedge feas)$ is unsatisfiable.*

The claim almost immediately follows from the definition of $\sigma-$monitorable properties, Claim 3, and Claim 4. Note that the algorithm can be very inefficient due to the double-exponential complexity. One way to cope with the problem is to consider each state in α_k and β_k individually. The idea is very similar to never-violate states introduced in [5]. A state of a Büchi automaton is called *never violate* if, on any input letter, there is a transition to another *never-violate* state. Similarly, we can define *never-satisfy* states and obtain a reasonable approximation to the problem of monitorability. Note that the complexity of this solution is exponential, which hopefully can be managed using BDD's. In addition, the never-violate and never-satisfy states can be pre-computed before the monitoring starts. However, it remains to be seen whether the approximation works well in practice.

11 Related Work

It is very interesting to compare our approach to the one suggested in [4], which uses alternating automata. We have already mentioned some high-level distinctions between testers and alternating automata in Section 4. However, the question remains about which construction is better. It turns out that both approaches yield very similar results, assuming universal non-determinism is removed from the alternating automata. Although that is a somewhat unexpected conclusion, it is not hard to justify it.

Without going into the details of algorithm described in [4], it is enough to mention that each state in the alternating automaton is essentially labeled with a sub-formula. To remove universal non-determinism, we follow classical subset construction. In particular, we assign a boolean variable x for each sub-formula φ to represent whether the corresponding state is in the subset. One can easily verify that x is nothing more but the output variable of the tester T_φ and follows the same transition relation.

To finish the partial determinization and define the final states in the new automata, the authors of [4] use the same trick with double representation as we do. At this step, the automata obtained after the subset construction is composed with itself via a cartesian product. This step is conceptually the same as introducing Y variables in the tester construction. However, we only introduce the extra variables when dealing with SERE's. For the LTL portion of the formula, the tester construction avoids the quadratic blow out associated with the cartesian product by essentially building a generalized Büchi with multiple acceptance sets (i.e., multiple justice requirements). If one to insist on a single acceptance set, our approach would yield an automaton identical to the one obtained in [4]. Note that, for symbolic model checking, using a generalized Büchi automaton might be more efficient then the corresponding Büchi automaton.

While our approach may not necessarily yield a better automaton, it never performs worse, and there are several significant benefits. Since model checking is very expensive, we expect that, in practice, automata for commonly occurring sub-properties will be hand-tuned. In such a case, it is more beneficial to work with testers since an alternating automaton requires an exponential blow-up due to universal non-determinism that cannot be locally optimized.

Another important advantage is that PSL testers can be used anywhere instead of LTL testers. For example, if one were to extend CTL* with PSL operators, our approach combined with [10] immediately gives a model checking algorithm for the new logic.

12 Conclusion

In this paper, we have shown a new approach towards model checking logic PSL, recently introduced as a new standard for specifying hardware properties. Our approach is based on testers that, unlike automata, are highly compositional, which is very advantageous in the context of PSL.

In addition, we have described a framework for symbolic run-time monitoring. In particular, we have identified some of the major questions that a good monitor should be able to answer and shown how to answer those questions using symbolic algorithms.

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A Associating a Regular Grammar with a SERE

Let b be a boolean expression, r', r, r_1, r_2 be SEREs, and $\mathcal{G}', \mathcal{G}, \mathcal{G}_1, \mathcal{G}_2$ the corresponding grammars. Our algorithm is recursive and we assume that $\mathcal{G}$, $\mathcal{G}_1$, and $\mathcal{G}_2$ have already been properly constructed. Our goal is to build $\mathcal{G}' = \langle \mathcal{V}', \mathcal{T}', \mathcal{P}', \mathcal{S}' \rangle$ for the SERE r'.

- $r' = b$
  - $\mathcal{V}' = \{V\}$
  - $\mathcal{T}' = \{b\}$
  - $\mathcal{P}' = \{V \rightarrow b\}$
  - $\mathcal{S}' = V$
- $r' = r_1 \,;\, r_2$
  - $\mathcal{V}' = \mathcal{V}_1 \cup \mathcal{V}_2$
  - $\mathcal{T}' = \mathcal{T}_1 \cup \mathcal{T}_2$
  - $\mathcal{P}' = \begin{array}{l} \{V \rightarrow aW \mid V \rightarrow aW \in \mathcal{P}_1\} \quad \cup \\ \{V \rightarrow a\mathcal{S}_2 \mid V \rightarrow a \in \mathcal{P}_1, a \neq \epsilon\} \quad \cup \\ \{V \rightarrow a\mathcal{S}_2 \mid V \rightarrow aW \in \mathcal{P}_1, W \rightarrow \epsilon \in \mathcal{P}_1\} \cup \\ \mathcal{P}_2 \end{array}$
  - $\mathcal{S}' = \mathcal{S}_1$
- $r' = r_1 : r_2$
  - $\mathcal{V}' = \mathcal{V}_1 \cup \mathcal{V}_2$
  - $\mathcal{T}' = \mathcal{T}_1 \cup \mathcal{T}_2$

- $\mathcal{P}' = \begin{array}{l} \{V \to aW \mid V \to aW \in \mathcal{P}_1\} \quad \cup \\ \{V \to a \wedge b \mid V \to a \in \mathcal{P}_1, \mathcal{S}_2 \to b \in \mathcal{P}_2\} \quad \cup \\ \{V \to (a \wedge b)W \mid V \to a \in \mathcal{P}_1, \mathcal{S}_2 \to bW \in \mathcal{P}_2\} \cup \\ \mathcal{P}_2 \end{array}$

  where $a \wedge b = \begin{cases} \epsilon, \text{ if } a = b = \epsilon \\ a, \text{ if } b = \epsilon \\ b, \text{ if } a = \epsilon \\ a \wedge b, \text{ otherwise} \end{cases}$

  - $\mathcal{S}' = \mathcal{S}_1$

- $r' = r_1 \mid r_2$
  - $\mathcal{V}' = \{\mathcal{S}'\} \cup \mathcal{V}_1 \cup \mathcal{V}_2$
  - $\mathcal{T}' = \mathcal{T}_1 \cup \mathcal{T}_2$
  - $\mathcal{P}' = \begin{array}{l} \{\mathcal{S}' \to aW \mid \mathcal{S}_1 \to aW \in \mathcal{P}_1\} \cup \\ \{\mathcal{S}' \to aW \mid \mathcal{S}_2 \to aW \in \mathcal{P}_1\} \cup \\ \mathcal{P}_1 \quad \cup \\ \mathcal{P}_2 \end{array}$
  - $\mathcal{S}' = \mathcal{S}'$

- $r' = r_1 \;\&\&\; r_2$
  - $\mathcal{V}' = \mathcal{V}_1 \times \mathcal{V}_2$
  - $\mathcal{T}' = \mathcal{T}_1 \cup \mathcal{T}_2$
  - $\mathcal{P}' = \begin{array}{l} \{(V, X) \to a \wedge b(W, Y) \mid V \to aW \in \mathcal{P}_1, X \to bY \in \mathcal{P}_2\} \cup \\ \{(V, X) \to a \wedge b \mid V \to a \in \mathcal{P}_1, X \to b \in \mathcal{P}_2\} \end{array}$
  - $\mathcal{S}' = (\mathcal{S}_1, \mathcal{S}_2)$

- $r' = [*0]$
  - $\mathcal{V}' = \{V\}$
  - $\mathcal{T}' = \{b\}$
  - $\mathcal{P}' = \{V \to \epsilon\}$
  - $\mathcal{S}' = V$
- $r' = r[*]$
  - $\mathcal{V}' = \mathcal{V}$
  - $\mathcal{T}' = \mathcal{T}$
  - $\mathcal{P}' = \begin{array}{l} \{\mathcal{S} \to \epsilon\} \quad \cup \\ \{V \to a\mathcal{S} \mid V \to a \in \mathcal{P}, a \neq \epsilon\} \quad \cup \\ \{V \to a\mathcal{S} \mid V \to aW \in \mathcal{P}, W \to \epsilon \in \mathcal{P}\} \end{array}$
  - $\mathcal{S}' = \mathcal{S}$