

A logical framework for commonsense predictions of solid object behaviour

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Predicting the behaviour of a qualitatively described system of solid objects requires a combination of geometrical, temporal, and physical reasoning. Methods based upon formulating and solving differential equations are not adequate for robust prediction, since the behaviour of a system over extended time may be much simpler than its behaviour over local time. This paper presents a first-order logic in which one can state simple physical problems and derive their solution deductively, without recourse to solving differential equations. This logic is substantially more expressive and powerful than any previous AI representational system in this domain.

Key Words: physical reasoning, spatial reasoning, qualitative physics.

1. INTRODUCTION

The behaviour of solid objects is an important and natural domain for commonsense reasoning. Solid objects are involved in most terrestrial natural phenomena and in nearly every man-made artifact. They are familiar to every human from infancy and are fairly well understood by childhood.

There are two major difficulties in building an AI system that reasons about solid objects. First, the physics of solid objects involves complex geometric, temporal, and physical reasoning. Many different aspects of geometry are relevant: the topology of objects, their gross shape and dimensions, and their differential surface characteristics. The motions even of simple physical systems can be bewildering; for example, gyroscopic motion is baffling when first encountered. Even with all the tools of mathematical and physical analysis, simple problems can be hard to solve; for example, the exact two-dimensional analysis of pushing a peg into a hole in the presence of friction is very difficult¹.

The second difficulty is that a useful AI system must degrade well under data limitations; it must be able to derive useful partial results from incomplete and imprecise problem specifications. An AI system must be able to produce a crude analysis even in cases where exact information about object shapes or parameters is unavailable or unspecified (as in the middle of a design process) or is too complex for effective use. An AI system must also be able to reason generically about classes of varying problems. For example, if we build a chicken fence, we want to reason that no chicken within the range of chicken shapes and sizes can go through the fence; we do not want to calculate this for each individual chicken. In this kind of qualitative reasoning, human common sense far exceeds conventional computational techniques. Our task is to create an AI system with the same ability to apply common sense to these problems.

This paper presents a logical framework for qualitative reasoning about solid objects. We will present the structure of a first order language L in which the physics

of solid objects can be described; we will define the semantics of L in terms of a formal model; and we will demonstrate the usefulness of L by showing that interesting problems can be solved qualitatively by inference from plausible axioms expressed in L . The language L is much more expressive and supports much richer inferences than any previous representation scheme in this domain.

In concentrating on the representation and formulation of knowledge, and postponing any questions of algorithms or control structure, we are following the methodology advocated by Hayes². Despite recent criticisms of this approach³, it still seems the most promising avenue for research in knowledge representation and common sense reasoning. However, our theory departs from Hayes' program in several respects. First, it is not a naive theory. Rather, we use the entire machinery of Newtonian mechanics, including concepts, such as total mechanical energy, that are sophisticated and have no commonsense analogue. We do this because it is very difficult to develop an alternative mechanics that is consistent and powerful and that does not entail absurd consequences. (McCloskey⁴ had made an interesting analysis of the theories of motion actually held by naive subjects.) Second, it is at least doubtful whether we have satisfied Hayes' dictum that obvious facts should have short proofs. This depends, of course, on what axioms you take as the starting point and what you consider short.

The mathematics used here is not 'qualitative' in the narrow sense⁵; that is, it does not represent quantities purely in terms of order relationships and constants. As we have argued elsewhere⁶, this is an unacceptably restricting limitation for many types of common sense reasoning. Our theory is qualitative in the sense that it can work with partially specified information.

In the remainder of the paper, we develop our logical theory, and we use it to solve variants of one particular kind of problem: predicting what happens when one or two dice are dropped inside a funnel (Fig. 1). We will show that different forms of this problem require a rich,

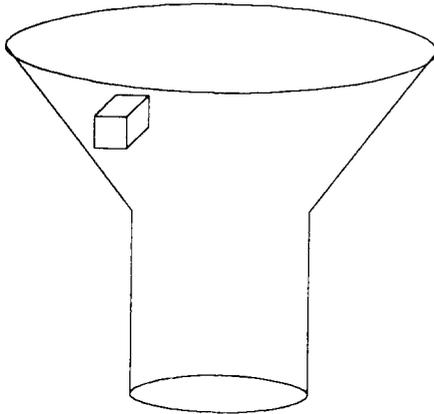


Fig. 1. A die is dropped inside a funnel

interconnected body of geometric and physical knowledge for their solution.

Section 2 deals with previous AI work in this domain. Section 3 presents several variants of the 'die in the funnel' example, and discusses how they can be solved in our theory. Section 4 discusses the geometrical, temporal, and physical theories which underly the theory. Sections 5, 6, and 7 are for true believers in formalism. Section 5 is a formal definition of the ontology of the language *L*. Section 6 presents the predicates used in solving one particular instance of the die in the funnel example. Section 7 carries out the analysis of this example. Section 8 gives our conclusions.

2. BACKGROUND

There has been a substantial body of AI research in reasoning about the physics of solid objects, Fahlman's BUILD program⁷ determined the stability of a tower of polyhedral blocks. De Kleer's NEWTON⁸ predicted the behaviour of a point mass sliding on a constraint. Bundy's MECHO⁹ used force analysis and conservation laws to make physical predictions in situations of specialized format. Forbus' FROB¹⁰ predicted the behaviour of a point mass flying among constraints. Funt's WHISPER¹¹ predicted the behaviour of a collection of objects by simulating it in a retina-like image. Shoham¹² has systematized the rules that govern the local mobility of an object within constraints. Recently, there have been several studies carried out in reasoning about the kinematics of mechanisms by Faltings^{13,14}, Gelsey¹⁵, and Joskowicz¹⁶. Nielson¹⁷ has begun work extending Falting's results to deal with forces and dynamics. Mason¹⁸, Peshkin and Sanderson¹⁹, and Wang²⁰ have analysed the behaviour of physical objects under semi-static and impact dynamics.

All these studies have provided valuable insights into the nature of physical reasoning about solid objects. However, this body of research as a whole suffers from two serious limitations. First, there has been very little work on physical reasoning using partial knowledge of the geometry of the objects involved. All of the works cited above except de Kleer's and Forbus' require an exact specification of the shapes of all the objects involved. The works of Mason, Peshkin and Sanderson, and Wang deal with ignorance of the positions of objects. The remaining studies require exact specification of position. De Kleer's NEWTON and Forbus' FROB

explicitly address the problem of partial shape and positional information, but they are very limited both in their spatial vocabulary, and in the range of physical interactions they address (moving objects are required to be point objects). Flexible representations of partial spatial information have been employed in computer vision²¹ and in cognitive mapping²²⁻²⁴, but not in physical reasoning.

Another limitation of these programs is more subtle, but more fundamental. Almost without exception, these programs are based on extrapolating behaviour over differential time. In order to make a prediction, the program first determines how each state of the system will tend to change, and then extrapolates these changes to predict a continual trend of change up to the point that the structure of the system changes. This extrapolation may be done either qualitatively, as in FROB and NEWTON, or symbolically, as in MECHO, or using point-by-point simulation, as in WHISPER, or numerically, as proposed by Gelsey (personal communication).

For example, FROB predicts the behaviour of a bouncing ball in a well by dividing the attainable physical space into 4 regions (the interior of the well, the bottom, and the two sides), and dividing the velocity space of the ball into nine (motionless, up, down, left, right, and the four quadrants.) (Fig. 2) There are thus 27 possible states of the system ($4 \times 9 - 9$ impossible states). The laws of physics are then used to determine which transitions between states are allowed, and thus a transition graph of states is developed. The final prediction is that the system follows a path in this transition graph.

However extrapolation is done, this kind of simulation is inherently inadequate for robust prediction in this domain. In any analysis of this kind, each different set of the boundary conditions that can apply in different situations must be considered as a different state. Each such state must be separately detected, categorized, and analysed, and the system's progress through these states must be systematically recorded. In many problems, however, such a categorization is very difficult, and entirely pointless. Consider the problem in Fig. 1; a small die is released inside a large steep funnel. Many states are possible in this system: the die may be in free-fall; it may be colliding or in continuous contact with the top or bottom part of the funnel on any of eight vertices, twelve sides, or six faces; it may be spinning, sliding, or rolling, up, down, or around the funnel. If we wish to determine which face ends up on top, we will have to trace the progress of the die through all these states. Such tracing is difficult. However, if all we wish to determine is that the die falls through the funnel, then all these intermediate states do not matter and it should not be necessary to trace them. To determine by any kind of simulation that

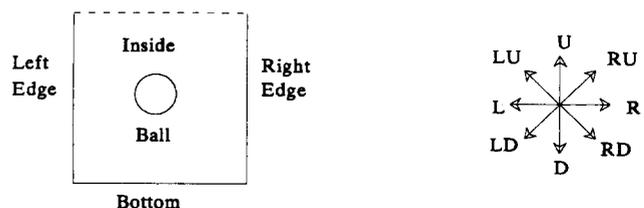


Fig. 2. Discretized space and velocity in FROB¹⁰

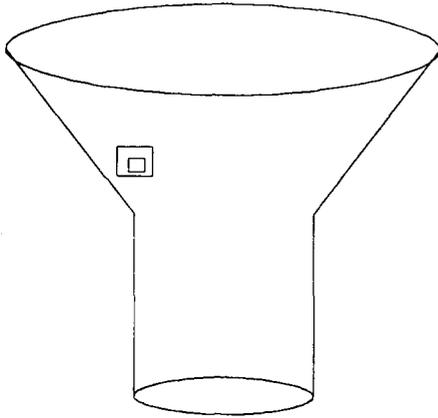


Fig. 3. One die inside another in a funnel

the die falls through the funnel will be almost as difficult as to determine how it lies. This seems implausible.

Two further arguments may be brought against simulation. First, in a problem like the die in the funnel, the sequence of states traversed and the final resting position of the die depend very delicately on the exact shapes, sizes, and physical properties of the die and the funnel. On the other hand, the conclusion that the die comes out the bottom is quite robust with respect to small variations in these parameters. Therefore, if the problem is specified with some small degree of imprecision, simulation will either be impossible, or will involve some monstrous tree of possibilities. But in qualitative reasoning we would like the prediction that the die comes out the bottom to be almost as easy with imprecise data as with precise. Second, the complexity of simulation goes up rapidly with the number of interacting objects. In Fig. 3, for example, with one die inside another dropped inside a funnel, the set of system states is the cross product of the possible interactions of the two dice with the possible interactions of the outer die and the funnel. Nonetheless, the prediction that the two dice will come out the bottom is almost as easy as in Fig. 1, with only one die.

The axiomatic analyses of mechanics developed by physicists such as Kilmister and Reeve²⁵ are similarly based on a differential analysis, and therefore cannot be used as the basis for commonsense physical reasoning. (Alternative formulations of mechanics based on variational principles, such as the principle of least action, are not better.)

In short, formulating and solving differential equations is an inadequate technique in this domain, since the behaviour of these physical systems over extended time is often easier to characterize than their behaviour over local time. A powerful physical reasoning system must be able to infer the general quality of a course of events from a broad characterization of the physical properties of the objects involved, without calculating each subevent.

The programs cited do use some techniques other than simulation. MECHO⁹ and NEWTON⁸ use energy conservation to prune possible system behaviours. Any state with more mechanical energy than the starting state can be ruled out as a possibility during the entire history of the system, regardless of any intermediate states. For example, in the 'die in the funnel' example, we may be sure that, whatever happens, the die will not come out the top of the funnel. FROB¹⁰ is able to predict that the system

must end up in a stable state, and therefore it must traverse a path in the state transition graph which ends in a stable state. The programs of Mason¹⁸, Peshkin and Sanderson¹⁹, and Wang²⁰ do not compute differential behaviour, but rather calculate transitions between global states. They do, however, consider every type of differential state attained by the system; owing to the strong assumptions made about the physics, there are only a small number of such types to consider. We believe that effective qualitative reasoning requires an increased use and development of principles of this sort, and a decreased reliance on simulation techniques.

A natural knowledge engineering approach to the problem would be to write down rules that simply state the desired prediction, such as 'A small object released inside a steep, large-mouthed funnel will fall out the bottom'. But rules of this kind are inadequate as any single rule of this kind can cover only a very small class of problems; to cover large classes of problems, we would need many separate rules. In particular, a rule like the one suggested above will apply only when the die and the funnel are the only objects involved. As soon as another object enters, the conclusion may not apply, and the rule gives no guidance. In technical terms, rules of this kind are not compositional across objects. Even when dealing with only a die and a funnel, if we allow wide ranges in the shape of these objects, the conclusion will sometimes apply and sometimes not. Since there is no simple, general rule for when a die will come out the bottom, many different rules, corresponding to special geometric cases, must be stated.

There are three problems with using many separate rules corresponding to special cases. First, in the long run it will lead to a very large knowledge base, and thus to space and time inefficiencies. Second, if a new case, not precisely covered in pre-determined categories, comes up, the system cannot even begin to deal with it. Third, this approach is aesthetically displeasing. A well-designed system should use similarities among different cases of a die falling through a funnel, and similarities between this problem and similar problems, such as a die shot through a tube, or a die dropped into a box. The analyses of these different cases ought to use much of the same fundamental knowledge.

3. EXAMPLES AND ANALYSIS

The 'die in the funnel' example can be analysed as follows: (i) Due to the topology of the funnel, if the die goes from inside it to outside it, it must either exit the top or exit the bottom. (ii) Since the die is dropped from rest inside the funnel, it cannot have the energy to exit the top of the funnel. (iii) There is no stable resting point for the die inside the funnel, since it is smaller than the funnel's mouth, and the funnel's sides are steep. (iv) The die cannot stay forever moving within the funnel, for its kinetic energy will eventually be dissipated. Therefore, the die must exit the bottom of the funnel. In many cases where common sense predicts that the die will come out the bottom, it is possible to carry out such an analysis and to support the substeps by inferences from rules of fairly broad applicability. (There are exceptions where this analysis is unnecessary and/or impossible; for instance, when the die is dropped from a great height directly above

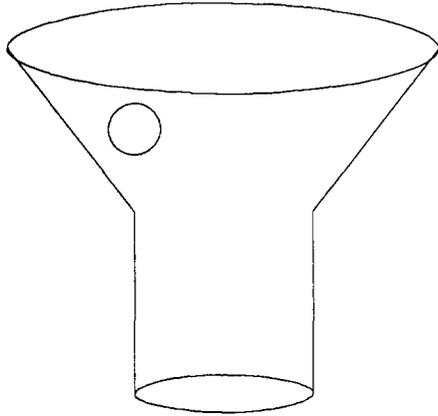


Fig. 4. Spherical die inside radially symmetric funnel

the mouth of the funnel.) Different categories of problems will vary in the justifications of the substeps.

Related problems will share parts of the analysis. For instance, in predicting that a die in a small-necked funnel will come to rest at the top of the neck, we may use the identical arguments (i) that the die must either exit the top, exit the bottom, or stay inside; (ii) that it cannot exit the top; and (iv) that it cannot stay inside in a perpetual state of motion. The argument (iii) that it cannot rest stably inside the funnel must be modified to an argument that it can only rest stably at the top of the neck of the funnel; and the additional argument must be made that it cannot exit the bottom of the funnel, since the orifice is too small.

This analysis avoids both problems discussed in section 2. We need not determine the differential states of motion of the die inside the funnel. Different categories of problems are analysed in similar ways from principles of broad utility. In the remainder of this section, we show how an analysis of this sort can be applied to various geometric specifications of the 'die in the funnel' example.

We begin with the simple case shown in Fig. 4. The die is a uniform sphere. The funnel is the surface of revolution about a vertical axis of a planar figure with a convex inner side. The radius of the die is less than the radius of revolution of the funnel. The steps of the argument are easily filled in. (i) The top and bottom of the funnel are the only orifices of free space connecting the inside of the funnel with its outside. Therefore, if the die is to go from inside to outside, it must go through the top or the bottom. (ii) Since the die is spherical, its centre of mass is in its interior. Since the top of the funnel is horizontal, and directed upward, if the die were to exit it, for each point in the interior of the die there would be an instant of time when it was above the top of the funnel. In particular, there would be an instant when the centre of mass would be above the top. But the die started out from rest below the top of the funnel, and there is no source of additional energy for the die. Therefore, the die cannot come out the top. (iii) Geometrically, the die can only be supported at the inside of the funnel in a single point. A rule of statics asserts that a uniform spherical object can be stably supported at a single point only if the supporting surface is horizontal at that point. But the inner surface of the funnel is nowhere horizontal. Hence there is no resting place for the die inside the funnel. The full details of this argument are given in sections 6 and 7.

3.1. Out the top, out the bottom, or stay inside

We now consider how the various parts of this argument can be generalized and modified. Part (i), that the die must either exit the top, exit the bottom, or stay inside, does not, of course, require that the funnel be a solid of revolution; it requires only that the funnel be a tube with only two orifices. We can weaken the condition still further, and require only that all orifices other than the top or the bottom be too small to let the die go through. In many special cases, it is easy to determine whether a die can go through a hole.

3.2. Not out the top

Part (ii), the argument from energy conservation that the die cannot come out the top, depends on the die being convex and on the centre of mass of the die starting out below any part of the top. Actually, the only reason we need convexity is to establish that the centre of mass of the object is in its interior. If we can do this otherwise – for example, by exact calculation, or by establishing that the shape of the object lies within some sufficiently small perturbation of a convex shape, and that the mass is not all concentrated in the perturbation – then that is sufficient.

We can weaken the requirement on the shape of the die still further to include shapes such as toruses and wiffle balls. We need to define two geometrical terms. A shape S is said to be *ringless* if any planar cross-section of S is simply connected (i.e., the cross-section has no holes in the middle). For example, a torus or a hollow sphere are not ringless (Fig. 5). We next define the *ringed filling-in* of a three-dimensional shape S as the minimal ringless shape containing S . It can be easily shown that if a point X is in the ringed filling-in of S , and S goes through a plane P , then X also goes through P . Therefore, if the centre of mass of the die lies within its ringed filling-in, then the die cannot come out the top of the funnel.

We can apply the same argument in cases where we have two or more dice bound together. If the dice are bound so that the union of their shapes or its ringed filling-in is always convex, then the same argument applies to the centre of mass of the pair. For instance, if one of the dice contains the other, and has a convex exterior, as in Fig. 3, then it will not come out the top.

However, the argument cannot be extended straightforwardly to boomerang shaped objects (Fig. 6). The centre of mass of a boomerang is inside its curve; a boomerang can come out the top of funnel while keeping the centre of mass always lower than the top. Nor does the argument apply if there are several dice which are not bound together. In Fig. 7, for example, it is possible that die B will eventually collide with die A in such a way that

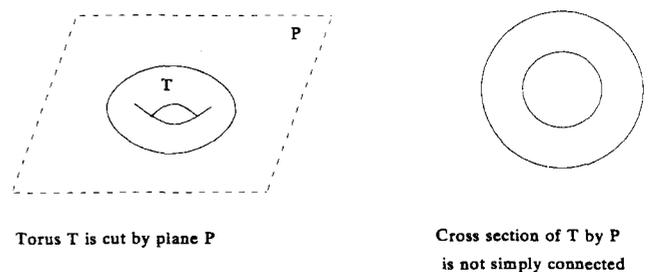


Fig. 5. Torus T is not ringless

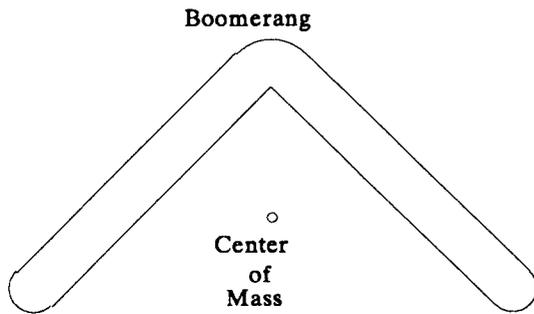


Fig. 6. Boomerang with centre of mass

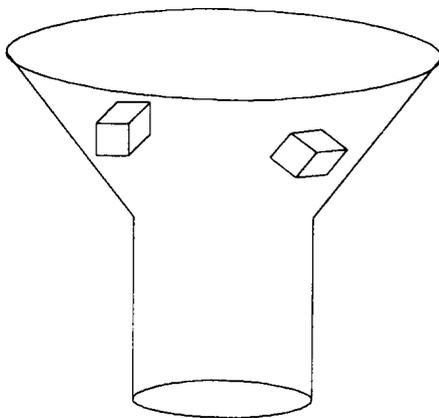
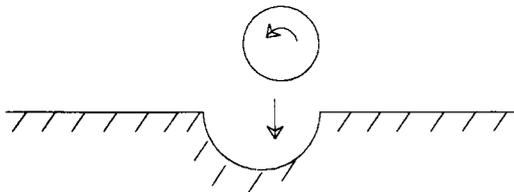


Fig. 7. Two dice in a funnel



The ball will spin around the dent without energy loss

Fig. 8. Collision of inelastic objects I

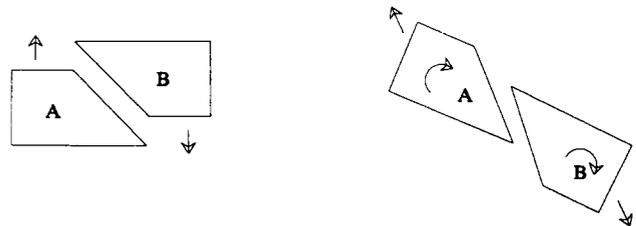
A gets enough energy to escape out the top. How can we rule out such cases?

In fact, if these objects collide elastically, then it is entirely plausible that the boomerang die, or one of the two dice dropped together, will come out the top. If two Ping Pong balls were dropped inside a funnel, it would not be in the least surprising if one came out the top. On the other hand, if the objects are made of an inelastic material, like wood, then intuitively it seems safe to predict that enough energy would be lost in the initial collisions to allow the objects to fly out. It is, however, hard to make this argument precise, because under certain conditions inelastic objects can collide without loss of energy. Consider the situation shown in Fig. 8. No impact takes place; hence, no energy is lost during impact. No energy is lost to the normal forces, since normal forces are nondissipative. Hence, in the absence of friction, no energy whatever is lost, and the ball will fly from the wall at the same velocity at which it struck the wall. If the ball comes in with the correct forespin, no energy will be lost in the presence of friction, either. Other anomalous cases which must be considered in formulating a general, useful rule of collisions over

extended time include Fig. 9, in which one inelastic object sideswipes another, with very little energy loss; and Fig. 10, in which a ball thrown up a slanting rooftop comes down again at very nearly the same speed. To justify our inference in these examples, we must therefore either find conditions that exclude all such anomalies, or formulate a probabilistic or default rule that such elastic collisions are extremely unlikely. Formulating such rules for reasoning about the net effect of collisions is one of the most important open problems in our theory.

3.3. No resting point inside

Part (iii) of our argument, that the die cannot rest inside the funnel, depended on the strong conditions that the die was a uniform sphere and that it could contact the funnel only in a single point. We can easily achieve a slight generalization to nearly uniform, nearly spherical dice. Let θ be the vertical angle of the support, let μ be the coefficient of friction; let ϕ be the maximum angle between the line from the centre of mass to a point on the surface and the normal at that point (Fig. 11). The ball can stand still only if $\mu \geq \tan(\theta)$ and $\phi > \theta$. A further generalization replaces the condition that the objects contact in a single point with the weaker condition that the surface normal at all contact points are parallel. Thus, we can use these formulas to predict that a regular icosahedral die will roll or slide down a sufficiently steep funnel.



Little energy lost in a sideswipe

Fig. 9. Collision of inelastic objects II

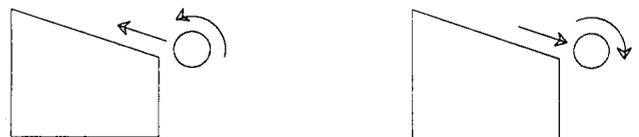
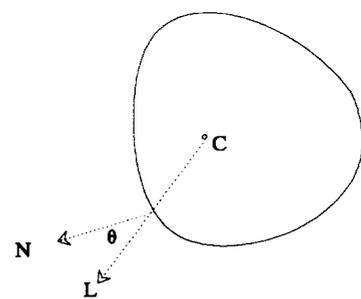


Fig. 10. Collision of inelastic objects III



C — center of mass
N — normal to the surface
L — line through C
 θ — angle between N and L

Fig. 11. Distorted sphere

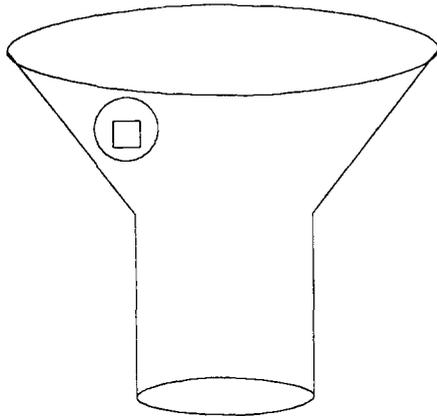


Fig. 12. Spherical die contains another die

We can similarly handle the case of a die which is a spherical shell and contains another die inside (Fig. 12). Such a system can rest statically only if there is a position in which joint centre of mass of the two dice is located directly above the contact point of the outer die with its support, and in which the inner die can rest stably inside the outer die. The conditions which determine this can either be taken to be high level axioms or derived from a force analysis.

With dice that can contact the funnel in several points with different surface normals, the analysis is sufficiently harder that we will forget about rolling and just look for conditions sufficient to produce sliding. The wider the range of the horizontal component of the surface normals at contact points, the steeper the slope must be, for the normal forces at the various contact points will tend to act against each other; hence, they will be larger than they would be otherwise; hence, the friction forces will be larger. The following rule holds: Let A be in contact with B . Let θ be the minimum vertical angle of the surface of B at a contact point. Consider the horizontal components of the surface normals of B at the contact points, and assume that there is some direction which lies within some small angle ϕ of all these horizontal components. (Fig. 13) Let the coefficient of friction be μ . If $\mu < \cos \phi \tan \theta$, then A will slide down B . This formula can be shown by an extension of the proof that a block on an inclined plane slides if $\mu < \tan \theta$.

The geometric condition that the surface normals at the contact points are all nearly parallel in their horizontal components can be established in various ways. For example, one can get this by bounding the diameter of the die, and the curvature of inner surface of the funnel; or by special formula for some class of shape description; etc.

The above formula will work just as well for multiple dice, if we consider all the contact points with the funnel of some collection of dice connected by abutments. Thus, we can easily show that if a collection of dice is so small that, strung end to end, they can only reach points on the funnel with surface normals with close horizontal components, then they must fall through. It is undoubtedly possible to establish stronger rules with a little more work.

Combining all the different ways in which the results (i), (ii), and (iii) may be established together and all the ways in which their geometrical preconditions may be satisfied gives a rich, interconnected body of results, all with the conclusion, 'The die falls out the bottom of the funnel'.

4. THE UNDERLYING LOGIC

We now address the central question of this paper: What logical framework, and what knowledge of geometry, temporal logic, and physics, are necessary to carry out the above reasoning?

4.1. Geometry

The geometry of the world is taken to be R^3 . The space occupied by an object in some particular position is a subset of R^3 ; it is assumed to be closed, connected, and normal. By 'normal', we mean that no part of the object is a two dimensional lamina, or a one dimensional curve; technically, the shape of the object is equal to the closure of its interior. The fact that objects are closed means that abutting objects actually share boundary points; though this seems a little strange, it does not cause any actual problems in the logic. (Fleck²⁶ proposes an interesting topology that avoids this anomaly.) We will further assume that the boundaries of objects are everywhere smooth, with a well-defined tangent plane. This assumption is not strictly necessary, but it simplifies the logic.

The arguments in section 3 used several different kinds of geometric reasoning. These included the following:

- The ability to name and describe particular point sets that are connected to objects and are useful for physical reasoning, such as the *top* and *bottom* of a funnel. These are called 'pseudo-objects'. The problem of choosing pseudo-objects that will be useful in analysis is similar to the problem of constructing metric diagrams in FROB¹⁰.
- The use of topological predicates. For example, the funnel forms, topologically, a box with two orifices, and the die starts inside the box. (Predicates of this kind are examined in detail in a previous paper²⁷.)
- The use of a property quantified over all irregularities of a certain kind in an object. For instance, we may wish to say that the funnel has no holes large enough to let the die through other than the top and bottom.
- The use of special shapes, such as spheres and surfaces of rotation, and the use of rules peculiar to these shapes.
- The use of metric dimensions, and the statement of inequalities on them. For example, we wish to say that the radius of the die is less than the radius of the funnel.
- The bounding of the range of the surface normal over a part of an object's surface. For example, we wish to say that the slope of the funnel is everywhere positive

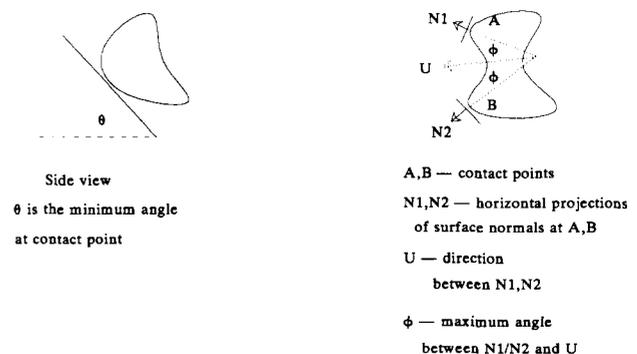


Fig. 13. General conditions for support

in its inner surface; or that, for each region of a specified size in the inner surface, there is a central direction such that the horizontal component of the surface normal lies within some small angle of the central direction.

- The use of convexity and related properties.

Ideally, we would like a systematic representation of shape and position that is capable of representing any three dimensional scene at a reasonable level of detail, that allows all the above properties to be expressed directly or easily inferred, and that supports the expression of many different kinds of partial information. We have not, to date, been able to come anywhere close to this ideal.

4.2. Temporal logic

Our temporal logic follows that of McDermott²⁸ fairly closely. A *scene* is an instantaneous snapshot of the universe. In our restricted domain, a scene consists precisely of a specification of the positions and velocities of all objects. A *chronicle* is a function from the time line to scenes. The time line is taken to be R^1 .

Unlike McDermott, we do not need the concept of branching time. However, it may be useful to use multiple chronicles for expressing physical principles in terms of counterfactuals e.g., ‘A bullet in free fall is always lower than it would be if it went in pure linear motion’. Note that the use of the counterfactual means that chronicles must range over physically impossible chains of events, as well as physically possible ones. (Counterfactual chronicles have been used by Shoham²⁹ and in my own previous work³⁰.)

In the examples of section 3, the only temporal predicates needed relate purely to order relationships. For other predictions, such as determining roughly *when* the die will come out the bottom of the funnel (in a second? in a minute? next year?) actual measurements on time must be used. We have not addressed this issue.

The ‘frame’ or ‘persistence’ problem of determining what remains true over time^{28,29,31} requires no special treatment in our logic. We avoid this problem, not through any special cleverness on our part, but by virtue of the structure of the domain. There are two types of predicates in this domain. The first type includes predicates which depend on position and velocity of objects. These are not assumed to remain constant over any interval unless they can be proven to be so. The second type includes structural predicates, depending only on the shape and other material properties of the objects. These are assumed to be constant over the problem, and so are defined atemporally. Similar considerations would seem to apply to any closed world, complete physical theory; it is not clear why the frame problem should ever create trouble in such a domain. (The closed world assumption is explicitly introduced in our theory through the predicate ‘isolated(OO, C)’, which asserts that, during C , no mobile object in the set of objects OO ever comes into contact with any object outside OO . See sections 6.3, axiom 1.6.)

4.3. Motion

Reasoning about motion combines spatial and temporal reasoning. This domain requires both differential characterizations of motion, such as ‘sliding along a surface’, and global characterizations, such as

‘exiting through a face’. The inferences required include the following:

- Characterizing the paths than an object can take without coming to overlap other objects.
- Predicting that a motion will eventually cause some pair of objects to come into contact or to lose contact.

4.4. Physics

Our worlds consist of a finite set of solid objects which move about in space through time. Objects are completely rigid, and indestructible; the set of existent objects always stays the same. The interior of objects may not overlap.

Objects are subject to four kinds of forces: a uniform downward gravitational force; normal forces, which act to prevent objects from overlapping; friction; and a weak drag force, which serves to dissipate energy of motion when no other force is available. The drag force is assumed to be very much smaller than any other force. We need to assume its existence in order to postulate that no system can persist indefinitely in a state of motion. Friction by itself will not do this for us; a rolling ball loses no energy to friction.

Certain objects are *fixed*. A fixed object does not move, no matter what forces are applied to it. One particular fixed object, which is part of every chronicle, is the *ground*. The ground is a fixed object of infinite horizontal extent; our model is weak on astronomy. Any vertical line intersects the surface of the ground in a unique point; all points below the surface are part of the ground.

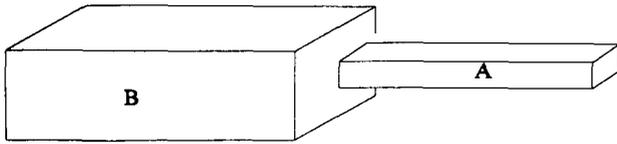
Objects have two other internal properties besides their shape: a distribution of mass within the interior, and a coefficient of elasticity, which determines how the object behaves in a collision. Any pair of objects have a coefficient of friction, which determines the frictive forces when the objects are in contact.

Thus a particular world is determined by enumerating the objects in the world with their shapes, mass distributions, coefficients of elasticity, and pairwise coefficients of friction. When a collision takes place, the velocity is discontinuous. By convention, we will define the velocity of an object in a collision scene as the limit of its velocity from previous times. (We assume, without proof, that this limit always exists.) Thus, we may specify as the initial conditions of a problem that a collision is taking place.

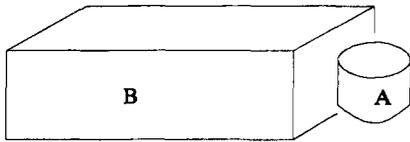
We need a fairly flexible ontology for forces. The problem is that one object may exert force on another at a single point, along a curve, or over a surface (Fig. 14). In simple physics problems, one generally considers the resolvable force as acting at a single point, but this is not feasible in the more general case, since the resolvable force may not be perpendicular to the surface there. The solution is to consider a force on an object as a distribution of force over the volume of the object. That is, given any subset of the object’s shape, a certain force is applied to the object over that subset.

The physical deductions which must be made in the examples of section 3 include the following:

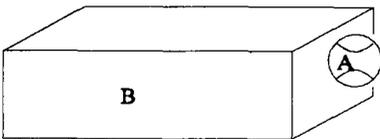
- Determining whether a set of objects can attain a stable scene while certain geometric conditions hold.
- Finding constraints on the location of the centre of mass of an object or a set of objects.



A pushes on B on a face.



A pushes on B along a line.



A pushes on B at a point.

Fig. 14. Force applied across a face, a line, or a point

- Resolving a set of forces, and determining motion under those forces.
- Predicting the effect of a collision.
- Determining whether a chronicle violates a conservation law, particularly conservation of energy.

5. ONTOLOGY

The remainder of this paper explicitly constructs a first-order language and a set of axioms adequate to analyse the first and simplest example of section 3. Section 5 constructs the formal ontology needed for solid object physics. Section 6 defines the nonlogical terms of the language and states the physics axioms needed to analyse the particular example. (The geometric and temporal axioms needed are given in a fuller version of this paper³². They have been omitted here because of limitations of space. Similarly, the proofs of the purely geometric lemmas 7.2.8, 7.2.9, and 7.2.12 may be found in the longer paper.) Section 7 carries out this analysis. We have not carried out an analysis at this level of detail for any other examples, but we believe that it can be done.

Our physical ontology contains numerous types of elements: instants of time, quantities of mass, quantities of energy, points, vectors, vector fields, mappings, movements, objects, scenes, pseudo-objects, chronicles, and sets of these. Unlike section 3, we will assume in these sections that all objects have uniform mass distributions. It would be easy to extend the logic so that this assumption can be made or dropped explicitly.

Time, mass, and energy are all measure spaces, which we may view as real numbers. It is not necessary to consider branching time in our domain.

We use R^3 as our underlying geometry. Points and vectors have their usual meaning. We will use the phrase 'geometric entity' to mean a point, a set of points, a vector, or a vector field. We specify changes in positions using *rigid mappings*; these are adequate, since we are dealing only with rigid objects. A rigid mapping is the composition of a translation and a rotation; reflections are not allowed. A *general velocity* is a derivative of a rigid mapping. A general velocity is the composition of a linear velocity and an angular velocity around a specified axis.

Definition 5.1: A point set is a subset of R^3 .

Definition 5.2: A vector field is a pair $\langle XX, f \rangle$, where XX is a point set and f is a function from XX onto vectors. We use the function 'base(P)' and 'value(P, X)' to recover the components of the vector field. If $P = \langle XX, f \rangle$ is a vector field, then $\text{base}(P) = XX$, and, for $X \in XX$, $\text{value}(P, X) = f(X)$.

Definition 5.3: A *mapping* is a sense-preserving rigid function from R^3 onto itself.

Definition 5.4: If X is a point or set of points in R^3 and M is a mapping, then $\text{image}(M, X) = M(X)$.

Definition 5.5: A *general velocity* is the derivative of a function from the reals onto mappings.

Definition 5.6: An *object* is a primitive entity.

Definition 5.7: The function 'shape(O)' maps an object O onto a subset of R^3 .

Definition 5.8: A *scene* is a function on a set of objects, which takes each object into an ordered pair of a mapping and a general velocity. For any object O and scene S , $\text{mapping}(S, O)$ is the mapping specified by S for object O ; $\text{prior-velocity}(S, O)$ is the general velocity specified by S for O .

Definition 5.9: For any scene S and object O , $\text{place}(O, S)$ is the point set occupied by O in S .

$$\text{place}(O, S) = \text{image}(\text{mapping}(S, O), \text{shape}(O))$$

Definition 5.10: A *pseudo-object* is an ordered pair of an object and a geometric entity. We write $Q = \text{pseudo-object}(O, XX)$.

Definition 5.11: Let $Q = \text{pseudo-object}(O, XX)$ be a pseudo-object. Then $\text{source}(Q) = O$, and $\text{shape}(Q) = XX$. For any scene S , $\text{place}(Q, S) = \text{image}(\text{mapping}(S, O), XX)$.

Definition 5.12: A *chronicle* is a function from an interval of time into scenes. For any chronicle C , $\text{duration}(C)$ is the domain of C , a time interval. For any C and any $T \in \text{duration}(C)$, $\text{scene}(C, T) = C(T)$, the scene of chronicle C at time T .

All chronicles are subject to the following constraints:

- The duration of the chronicle is closed on the left. Each chronicle has a starting moment.
- Any two scenes in the range of the chronicle have the same objects in their domain.
- For any object O in the chronicle, the function $\lambda(T) \text{ mapping}(\text{scene}(C, T), O)$ is continuous. That is, objects move continuously in space.
- For any object O in the chronicle, the function $\lambda(T) \text{ prior-velocity}(\text{scene}(C, T), O)$ is continuous from the left.
- For any object O in the chronicle, the function $\lambda(T) \text{ prior-velocity}(\text{scene}(C, T), O)$ is equal to the derivative from the left of the function $\lambda(T) \text{ mapping}(\text{scene}(C, T), O)$, at all T after the start

of the chronicle. That is, the velocity is the derivative of the position.

Note that chronicles do not have to be physically possible. We will use the predicate 'phys-poss(C)' to distinguish chronicles that obey the laws of physics.

6. AXIOMS FOR PHYSICAL REASONING

In this section we develop a first order language and physical axioms adequate to solve the first 'die in the funnel' example. All rules will be stated, first in English, then in first-order logic. We use a sorted logic, indicating the sort of a variable by its first letter. The following sorts are used, with the indicated keys.

Sort	First letter
Instant of time	T
Point in R^3	X
Vector	V
Vector field	P
Mapping	M
Object	O
Pseudo-object	Q
Scene	S
Chronicle	C

Sets of entities will be designated by a double initial letters. For example, variables denoting intervals of time will begin TT; variables denoting sets of objects will begin OO; etc. On occasion it will be useful to state axioms which can apply either to objects or to pseudo-objects; variables of this kind will begin with Q.

Capital letters are used for variables; lower-case letters for predicates, functions, and constants. Variables not explicitly quantified are assumed to be universally quantified, with the entire formula as scope.

The nonlogical terms and axioms of set theory and real analysis are assumed. (Only simple sets of the above sorts are used; hence, only the axioms on the boolean set operations are needed.) In our English definitions of functions and predicates, we will assume common geometrical concepts and vector algebra. Note that the only definitions and axioms listed are those we need for our particular example.

6.1. Geometry

6.1.1. Basic concepts

Function: 'distance($X1, X2$)' maps two points $X1$ and $X2$ onto the distance between them. In the usual way, we extend the distance function to take closed point sets as arguments, meaning the minimum distance between any points in them.

Predicate: 'beyond($X1, X2, V$)' (read ' $X1$ is beyond $X2$ in direction V ') holds for points $X1, X2$ and vector V if $(X1 - X2) \cdot V$ is positive. Applied to point sets, in the form 'beyond($XX1, XX2, V$)' it means that every point of $XX1$ is beyond every point of $XX2$.

Function: 'make-line(X, V)' is the line through point X parallel to vector V .

Predicate: 'single-point(XX)' holds if point set XX consists of a single point.

Function: 'interior(XX)' maps a point set XX onto its interior. 'Interior' is interpreted in the sense of a manifold;

a point $X \in XX$ is in the interior of XX if some neighbourhood of X in XX is homeomorphic to R^k for some k . Thus, lines, surfaces, and solids all have their own natural sense of interior.

Function: 'boundary(XX)' maps a point set XX onto its boundary. The boundary of XX is equal to its closure minus its interior (as defined above).

Predicate: 'body(XX)' means that XX is the closure of a connected, nonnull, three dimensional manifold.

Predicate: 'surface(XX)' means that XX is the closure of a connected, nonnull, two dimensional manifold.

Predicate: 'abut($XX1, XX2, XXXA$)' is true if $XX1$ and $XX2$ intersect only in their boundaries, and $XXXA$ is their intersection.

Predicate: 'overlap($XX1, XX2$)' holds of point sets $XX1, XX2$ if their interiors intersect.

Predicate: 'convex(XX)' means that the point set XX is convex in the usual sense: any two points in XX can be connected by a line segment lying in XX .

Function: 'convex-hull(XX)' maps a point set XX to its convex hull.

Function: 'diameter(XX)' maps a point set XX onto the maximum distance between any two points.

Predicate: 'bounded(XX, V)' means that point set XX is bounded in direction V .

Predicate: 'extreme(X, XX, V)' means that X is an extreme point of XX in direction V .

6.1.2. Planar geometry

Predicate: 'planar(XX)' holds if point set XX lies in a plane.

Predicate: 'cross-section(XXC, XXB, XXP)' means that XXC is a cross section of body XXB by plane XXP . Formally, XXC is the closure of a connected component of the intersection of the interior of body XXO with plane XXP . (This convoluted definition is necessary to handle cases where plane cuts a body in several surfaces, where a plane is tangent to a body, or, worse, where it is tangent for a while and then enters the body.)

Function: 'top(XX, V)' maps a closed, bounded, point set XX and a vector V into the plane perpendicular to V passing through the extreme point(s) of XX in the V direction. The function 'bottom(XX, V)' is the same in the $-V$ direction.

Predicate: 'side(XXS, XXP)' holds for point sets XXS and XXP if XXP is a planar surface and XXS is a connected subset of the boundary of XXP .

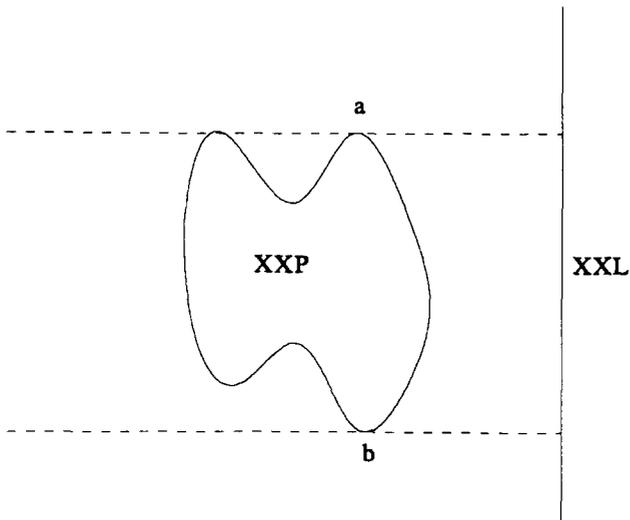
Predicate: 'd-side(PC, XX)' (read ' PC is a directed side of XX ') is true if XX is a planar surface, and PC is the vector field of the normal to XX along some side of XX .

Predicate: 'convex-side(P, XX)' holds for a vector field P if P is the directed side of XX and also of some convex shape.

Function: 'inner-d-side(XXP, XXL)' takes as arguments a planar object XXP and a disjoint line XXL in the same plane. Its value is defined as follows: Drop the perpendiculars from the top and bottom of XXP to XXL . These divide the boundary of XXP into at least two parts. The part which lies closest to XXL is the inner side. (Fig. 15)

6.1.3. Surface characteristics

Predicate: 'face(XXF, XXO)' holds for point-sets XXF and XXO if XXO is a body, and XXF is a connected subset of boundary(XXO).



The inner side of **XXP** with respect to **XXL** is the part of its boundary between **a** and **b**.

Fig. 15. Inner side

Predicate: 'smooth(XX, X)' is true if XX is a body, X is in the boundary of XX , and the boundary of XX is smooth at X .

Predicate: 'ordinary(XX)' is true if XX is a body which is everywhere smooth.

Function: 'surf-norm(XX, X)' gives the unit surface normal to a body XX at a point X on the boundary of XX . surf-norm(XX, X) is defined only if XX is smooth at X . surf-norm(XX, X) is directed out of XX .

Function: 'const-face(XX, V)' maps a planar shape XX and a vector V into the vector field equal to V at every point of XX .

6.1.4. Topology

Predicate: 'box(XX)' holds if XX is a bounded ordinary point set and the complement of XX has exactly two connected components.

Function: 'inside(XX)' and 'outside(XX)' are defined if XX is a box, and denote the inside and outside of the box. Technically, inside(XX) is the closure of the finite component of the complement of XX ; outside(XX) is the closure of the infinite component of the complement of XX .

Function: 'inner-boundary(XX)' and 'outer-boundary(XX)' are defined if XX is a box. They denote the inner and outer boundaries of XX .

Predicate: 'orifice(XXO, XXB)' holds if XXO is a body, XXB is a box, and XXO is a connected subset of XXB which abuts both the inside and outside of XXB .

Predicate: 'tube($XXT, P1, P2$)' (read ' XXT is a tube with caps $P1, P2$ ') if $P1$ and $P2$ are directed faces which 'close off' XXT into a box. Formally, $P1$ and $P2$ are the directed faces of two disjoint solids $XXC1$ and $XXC2$ which, together with XXT , form a box XXB in such a way that each of $XXC1, XXC2$, and XXT is an orifice of the box. (Fig. 16)

Function: 'tube-inside($XXT, P1, P2$)' applied to a tube XXT with caps $P1, P2$ gives the (closed) volume inside the tube defined by $XXT, P1, P2$.

Function: 'inner-surface($XXT, P1, P2$)' maps a tube XXT with caps $P1, P2$ into the part of the boundary of XXT on the inside.

Predicate: 'p-tube($XXT, P1, P2, V$)' holds if XXT is a tube with caps $P1, P2$, where $P1$ and $P2$ lie in the top/bottom of XXT in direction V , and have vectors equal to $V/-V$.

6.1.5. Special shapes

Predicate: 'circle(XX)' means that XX is a circle.

Predicate: 'sphere(XX)' means that XX is a sphere.

Function: 'radius(XX)' is defined if XX is a sphere, and gives its radius. The function 'centre(XX)' is defined if XX is a sphere, and gives its centre.

Function: 'solid-of-revolution(XXF, XXL)' is defined if XXL is a line; it is the solid swept out rotating XXF around XXL .

Predicate: 's-tube(XX)' (read ' XX is an s-tube') holds if XX is a body and XX is a solid of revolution of XXF around XXL where XXF does not intersect XXL .

Function: 'centre-line(XX)' is defined if XX is an s-tube, and returns the unique line around which XX is formed.

Function: 's-tube-top(XXF, V)' takes as arguments an s-tube XXF and a vector V that lies along the axis of XXF . It denotes a cap of XXF that lies in the top plane of XXF in direction V . The function 's-tube-bot(XXF, V)' is analogous. The function 's-tube-inside(XXF)' gives the tube-inside of XXF with respect to the top and the bottom.

Constant: 'vup' is the unit vector in the upward direction.

Function: 'height(X)' maps a point X into its height.

6.2. Chronicles, motions, and pseudo-objects

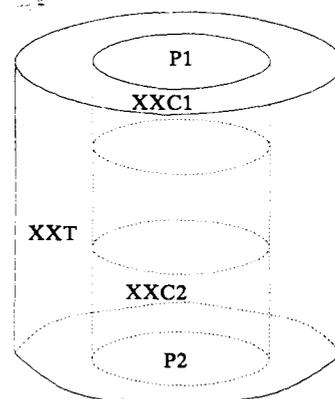
Function: 'scenes(C)' maps a chronicle C onto the set of scenes in C .

Function: 'startscene(C)' maps a chronicle C onto the first scene in C .

Function: The function 'subchronicle(C, TT)' maps a chronicle C and a time interval TT within the duration of C into the portion of C occurring during the interval TT .

Predicate: 'tail($C1, C2$)' means that $C1$ is the ending part of $C2$.

Predicate: 'infinite(C)' means that C extends infinitely into the future.



XXT — outer shell
P1, P2 — upper and lower faces
XXC1, XXC2 — top and bottom thirds of the cylinder

Fig. 16. Definition of a tube

Predicate: 'standard-position(O, S)' holds if mapping(S, O) is the identity.

Predicate: 'motionless(O, S)' means that object O has zero velocity in scene S .

Predicate: 'steady-state(C)' means that everything is motionless in C .

Predicate: 'exits(Q, QF, C)' means that (pseudo-)object Q goes outward through directed face QF during chronicle C .

6.3. Physics

6.3.1. Basic terms

Definition 1.1: The function 'centre-mass(O)' is the centre of mass (a pseudo-object) of an object O .

$$\text{source-of}(\text{centre-mass}(O)) = O$$

Axiom 1.2: The centre of mass of an object is inside its convex hull.

$$\text{shape}(\text{centre-mass}(O)) \in \text{interior}(\text{convex-hull}(\text{shape}(O)))$$

Definition 1.3: The function 'mass(O)' maps an object O into its mass.

Axiom 1.4: All objects has positive mass.

$$\text{mass}(O) > 0$$

Definition 1.5: The predicate 'fixed(O)' means that object O is considered a fixed obstacle. The opposite is mobile(O).

$$\text{mobile}(O) \Leftrightarrow \text{not}(\text{fixed}(O))$$

$$\text{fixed}(O) \Rightarrow \text{motionless}(O, S)$$

Definition 1.6: The predicate 'isolated(OO, C)' means that no mobile object in the set of objects OO comes into contact with any object outside OO in the course of C .

$$\begin{aligned} \text{isolated}(OO, C) \Leftrightarrow \\ [\forall_{O \in OO} \text{mobile}(O) \Rightarrow \\ [\forall_{O2, S, XX} [S \in \text{scenes}(C) \wedge \\ \text{abut}(\text{place}(O, S), \text{place}(O2, S), XX)] \Rightarrow \\ O2 \in OO]] \end{aligned}$$

Definition 1.7: The constant 'oground' is the ground.

Axiom 1.8: The ground is fixed in standard position in any chronicle.

$$\text{fixed}(\text{oground}) \wedge \text{standard-position}(\text{oground}, S)$$

Axiom 1.9: All objects have ordinary shapes.

$$\text{ordinary}(\text{shape}(O))$$

Axiom 1.10: The ground is the only unbounded object.

$$O \neq \text{oground} \Rightarrow \text{bounded}(\text{shape}(O), V)$$

6.3.2. Energy

Definition 2.1: The function 'energy(OO, S)' maps a set of objects OO in a scene S into the total mechanical energy of the objects. The function 'potential-energy(OO, S)' gives the potential energy. The function 'kinetic-energy(OO, S)' gives the kinetic energy.

Axiom 2.2: The total energy is the potential energy plus the kinetic energy.

$$\begin{aligned} \text{energy}(OO, S) = & \text{potential-energy}(OO, S) \\ & + \text{kinetic-energy}(OO, S) \end{aligned}$$

Axiom 2.3: The total energy of two disjoint sets of objects is equal to the sum of their separate energies

$$\begin{aligned} OO1 \cap OO2 = \emptyset \Rightarrow & \text{energy}(OO1 \cup OO2) \\ = & \text{energy}(OO1) + \text{energy}(OO2) \end{aligned}$$

Axiom 2.4: The potential energy of a single object is the height of its centre of mass times its mass times the constant g .

$$\begin{aligned} \text{potential-energy}(\{O\}, S) \\ = & \text{height}(\text{place}(\text{centre-mass}(O), S)) \cdot \text{mass}(O) \cdot g \end{aligned}$$

Axiom 2.5: The constant g is positive.

$$g > 0$$

Axiom 2.6: The kinetic energy of a motionless object is zero.

$$\text{motionless}(O, S) \Rightarrow \text{kinetic-energy}(\{O\}, S) = 0$$

Axiom 2.7: Kinetic energy is nonnegative.

$$\text{kinetic-energy}(OO) \geq 0$$

6.3.3. Stable scenes

Definition 3.1: The predicate 'possibly-stable(S)' means that S is a possibly stable scene. (The stability of an arrangement of objects may depend on the force distribution, which is, in general, underdetermined by the scene.)

Definition 3.2: The predicate 'possibly-supported(O, S)' (read 'object O is possibly supported in scene S ') is true if it is the case that, were every object except O held fixed in S , then O would definitely not move.

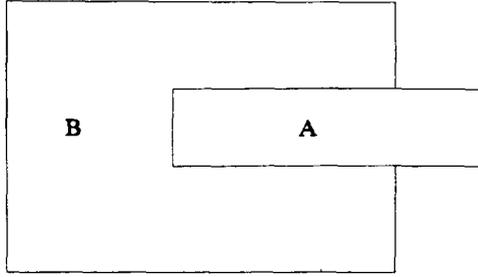
Axiom 3.3: If a scene is possibly stable, then every mobile object is possibly supported. (The converse are not true. See Fig. 17.)

$$\begin{aligned} [\text{possibly-stable}(S) \wedge \text{mobile}(O)] \\ \Rightarrow \text{possibly-supported}(O, S) \end{aligned}$$

Axiom 3.4: An object is not possibly supported unless it has some point of contact with another object where its surface has a nonpositive upward component.

$$\begin{aligned} \text{possibly-supported}(O, S) \Rightarrow \\ \exists_{O2, XXA, X} [\text{abut}(\text{place}(O, S), \text{place}(O2, S), XXA) \wedge \\ X \in XXA \wedge \text{surf-norm}(\text{place}(O, S), X) \cdot \text{vup} \leq 0] \end{aligned}$$

Axiom 3.5: If a spherical object abuts a single other object at a single point, then it is supported just if it has a downward surface normal at the point. (Note that this axiom is the only one in our system which rests on the assumption of uniform mass distribution.)



A and B are each "possibly supported".

(If either were fixed, it would support the other.)

However, if they are both mobile, they will fall together

Fig. 17. Possibly supported but not stable

$$\begin{aligned}
 & [\text{sphere}(\text{shape}(O1)) \wedge \\
 & [\forall_{OP,XXA} \text{abut}(\text{place}(O1,S),\text{place}(OP,S),XXA) \Leftrightarrow \\
 & [OP=O2 \wedge XXA = \{XA\}]]] \Rightarrow \\
 & [\text{possibly-supported}(O1,S) \Leftrightarrow \\
 & \text{surf-norm}(\text{place}(OP,S),XA) = -\text{vup}]
 \end{aligned}$$

6.3.4. Physically possible chronicles

Definition 4.1. The predicate 'phys-poss(C)' means that chronicle C is physically possible.

Axiom 4.2: Any subchronicle of a physically possible chronicle is itself physically possible.

$$\begin{aligned}
 & \text{phys-poss}(C1) \wedge \\
 & C2 = \text{subchronicle}(C1, T) \Rightarrow \text{phys-poss}(C2)
 \end{aligned}$$

Axiom 4.3: No two objects overlap in a physically possible chronicle.

$$\begin{aligned}
 & [\text{phys-poss}(C) \wedge S \in \text{scenes}(C)] \Rightarrow \\
 & \text{not}(\text{overlap}(\text{place}(O1,S),\text{place}(O2,S)))
 \end{aligned}$$

Axiom 4.4: The energy of an isolated system never increases.

$$\begin{aligned}
 & [\text{phys-poss}(C) \wedge \text{isolated}(OO,C) \wedge T1 < T2] \Rightarrow \\
 & \text{energy}(OO,\text{scene}(C,T1)) \geq \text{energy}(OO,\text{scene}(C,T2))
 \end{aligned}$$

Axiom 4.5: An infinite chronicle always ends in a steady state.

$$\begin{aligned}
 & [\text{phys-poss}(C) \text{ and } \text{infinite}(C)] \Rightarrow \\
 & \exists_{C1} \text{tail}(C1,C) \text{ and } \text{steady-state}(C1)
 \end{aligned}$$

Axiom 4.6: In a steady-state chronicle, every scene is possibly stable.

$$\begin{aligned}
 & [\text{phys-poss}(C) \text{ and } \text{steady-state}(C) \text{ and} \\
 & S \in \text{scenes}(C)] \Rightarrow \\
 & \text{possibly-stable}(S)
 \end{aligned}$$

7. FORMAL ANALYSIS

7.1. Problem statement

Consider a spherical die, and a radially symmetric funnel. Assume that the inner radius of the funnel is greater than the radius of the die; and that the inner side of a radial cross section of the funnel is convex. If the die is

released inside the funnel, and the funnel is held fixed far from the ground, then the die will eventually fall out the bottom of the funnel.

Constants of the examples:

- odie – the die
- ofunnel – the funnel
- c – the chronicle
- xx-pfunnel – the planar form from which the funnel is generated
- xx-centre-line – the axis of the funnel
- xcentre – a point on the axis of the funnel

Assumption 7.1.1: The die is a sphere.

$$\text{sphere}(\text{shape}(\text{odie}))$$

Assumption 7.1.2: The die is not fixed.

$$\text{mobile}(\text{odie})$$

Assumption 7.1.3: The funnel is the solid of revolution of xx-pfunnel around xx-centre-line.

$$\begin{aligned}
 & \text{shape}(\text{ofunnel}) \\
 & = \text{solid-of-revolution}(\text{xx-pfunnel}, \text{xx-centre-line})
 \end{aligned}$$

Assumption 7.1.4: xx-pfunnel is a radial cross section of the funnel.

$$\text{planar}(\text{xx-pfunnel} \cup \text{xx-centre-line})$$

Assumption 7.1.5: The inner boundary of xx-pfunnel with respect to xx-centre-line is convex.

$$\begin{aligned}
 & \text{convex-side}(\text{inner-dside}(\text{xx-pfunnel}, \\
 & \text{xx-centreline}), \text{xx-pfunnel})
 \end{aligned}$$

Assumption 7.1.6: The radius of the funnel is greater than the radius of the die.

$$\text{distance}(\text{xx-pfunnel}, \text{xx-centreline}) > \text{radius}(\text{odie}) > 0$$

Assumption 7.1.7: The axis of the funnel is vertical.

$$\text{xx-centreline} = \text{make-line}(\text{xcentre}, \text{vup})$$

Assumption 7.1.8: The funnel is oriented in standard position.

$$\text{standard-position}(\text{ofunnel}, \text{startscene}(c))$$

Assumption 7.1.9: The funnel is fixed.

$$\text{fixed}(\text{ofunnel})$$

Assumption 7.1.10: The die is isolated from everything but the funnel and the ground*.

$$\text{isolated}(\{\text{odie}, \text{ofunnel}, \text{oground}\}, c)$$

* We need the ground, because otherwise the hypotheses are inconsistent with the axioms. The axioms assert that an infinite chronicle must come to an end in a steady state. Since we will show that there is no steady state for the die inside the funnel, we must provide it with the ground to rest on.

Assumption 7.1.11: The funnel is more than the diameter of the die above the ground.

$$XF \in \text{shape}(\text{ofunnel}) \wedge XG \in \text{shape}(\text{oground}) \Rightarrow \text{height}(XF) - \text{height}(XG) > \text{diameter}(\text{odie})$$

Assumption 7.1.12: The chronicle is eternal.

$$\text{infinite}(c)$$

Assumption 7.1.13: The chronicle is physically possible.

$$\text{phys-poss}(c)$$

Assumption 7.1.14: The die starts from rest.

$$\text{motionless}(\text{odie}, \text{startscene}(c))$$

Assumption 7.1.15: The die starts from inside the funnel.

$$\begin{aligned} \text{place}(\text{odie}, \text{startscene}(c)) \subset \\ \text{tube-inside}(\text{shape}(\text{ofunnel}), \\ \text{s-tube-top}(\text{shape}(\text{ofunnel})), \\ \text{s-tube-bot}(\text{shape}(\text{ofunnel}))) \end{aligned}$$

Prove: The die exits the bottom of the funnel.

$$\begin{aligned} \text{exits}(\text{odie}, \text{pseudo-object}(\text{ofunnel}, \\ \text{s-tube-bot}(\text{ofunnel}, \text{vup})), c) \end{aligned}$$

7.2. Analysis

Lemma 7.2.1: An object inside a tube must eventually either find a stable state in the tube, or exit one of the faces.

$$\begin{aligned} & [\text{phys-poss}(C) \wedge \text{tube}(\text{shape}(OT), \\ & \text{shape}(QTOP), \text{shape}(QBOT)) \wedge \\ & \text{source}(QTOP) = \text{source}(QBOT) = OT \wedge \\ & S1 = \text{startscene}(C) \wedge \text{infinite}(C)] \Rightarrow \\ & [\text{exits}(O, QTOP, C) \vee \text{exits}(O, QBOT, C) \vee \\ & [\exists S \in \text{scenes}(C) \wedge \text{possibly-stable}(S) \wedge \\ & \text{not}(\text{disjoint}(\text{place}(O, S), \text{tube-inside}(\text{place}(OT, S), \\ & \text{place}(QTOP, S), \text{place}(QBOT, S))))]] \end{aligned}$$

Proof: Since C is infinite and physically possible, it must end in some stable state (4.5). If the object ever comes outside, then, since it cannot overlap the tube (4.3), it must exit either the top or bottom of the tube. If it never comes outside, then it is not outside in the stable state; that is, it is not disjoint from the inside.

Lemma 7.2.2: The energy of an isolated object O never increases.

$$\begin{aligned} & [\text{isolated}(OO, C) \wedge S \in \text{scenes}(C) \wedge \text{phys-poss}(C) \wedge \\ & \forall O1 \in OO [\text{mobile}(O1) \Rightarrow O1 = O]] \Rightarrow \\ & \text{energy}(\{O\}, S) \leq \text{energy}(\{O\}, \text{startscene}(C)) \end{aligned}$$

Proof: Since the set OO is isolated in C , its energy never increases (4.4). All the objects in OO other than O are fixed; that is, they are always motionless (1.5) and in the same place. Their kinetic energy is thus always zero (2.6) and their potential energy always constant (2.4, 1.1), so their total energy remains constant (2.2). Thus O is the only object whose energy may change. Since the energy of a set is the sum of the individual energies (2.3), the energy of O never increases.

Lemma 7.2.3: An isolated object O starting from rest never has its centre of mass higher than in the starting scene.

$$\begin{aligned} & [\text{isolated}(OO, C) \wedge S \in \text{scenes}(C) \wedge \text{phys-poss}(C) \wedge \\ & \forall O1 \in OO [\text{mobile}(O1) \Rightarrow O1 = O] \wedge \\ & \text{motionless}(O, \text{startscene}(C))] \Rightarrow \\ & \text{height}(\text{place}(\text{centre-mass}(O), S)) \leq \\ & \text{height}(\text{place}(\text{centre-mass}(O), \text{startscene}(C))) \end{aligned}$$

Proof: The energy of O cannot increase (7.2.2). Since it is motionless in the starting scene, its total energy to its potential energy (2.2, 2.6), which is an increasing function of the height of its centre of mass (2.4, 2.5, 1.5). If the centre of mass were to become higher, this would create a higher potential energy. Since kinetic-energy is nonnegative (2.7), the total energy would likewise be higher (2.2), but this is a contradiction.

Lemma 7.2.4: An isolated convex object which starts from rest below a fixed upward vertical face cannot exit through that face.

$$\begin{aligned} & [\text{isolated}(OO, C) \wedge \text{phys-poss}(C) \wedge \\ & \forall O1 \in OO [\text{mobile}(O1) \Rightarrow O1 = O] \wedge \\ & \text{motionless}(O, \text{startscene}(C)) \wedge \text{convex}(O) \wedge \\ & \text{shape}(QV) = \text{const-face}(XXV, \text{vup}) \wedge \\ & \text{fixed}(\text{source}(QV)) \wedge \\ & \text{standard-position}(\text{source}(QV), \text{startscene}(C)) \wedge \\ & \text{beyond}(\text{place}(QV, \text{startscene}(C)), \\ & \text{place}(O, \text{startscene}(C))m, \text{vup})] \Rightarrow \\ & \text{not}(\text{exits}(O, QV, C)) \end{aligned}$$

Proof: Since O is convex, the centre of mass of O is inside O (1.2). Since the face QV is beyond O in the starting scene (assumption), every point in QV is higher than the centre of mass of O . If O were to exit QV in C , then each point of O would have as high as QV in some scene of C . In particular, the centre of mass of O would have to be as high as QV , which is higher than the starting position of the centre of mass of O (assumption). But this is impossible by lemma 7.2.3. Thus, O does not exit QV in C .

Lemma 7.2.5: The funnel in our example is an s-tube.

$$\text{s-tube}(\text{shape}(\text{ofunnel}))$$

Proof: The funnel is the surface of revolution of xx-pfunnel around xx-centre-line (7.1.3). xx-pfunnel is some finite distance from xx-centre-line (7.1.6); hence they do not overlap. Therefore, the funnel is an s-tube.

Lemma 7.2.6: In our example, odie does not exit the top ofunnel in c .

$$\begin{aligned} & \text{not}(\text{exits}(\text{odie}, \text{pseudo-object}(\text{ofunnel}, \\ & \text{s-tube-top}(\text{ofunnel}, \text{vup})), c) \end{aligned}$$

Proof: We must show that the conditions in (7.2.4) hold, with $C=c$, $OO=\{\text{odie}, \text{ofunnel}, \text{oground}\}$, $O=\text{ofunnel}$, and $QV=\text{pseudo-object}(\text{ofunnel}, \text{s-tube-top}(\text{ofunnel}, \text{vup}))$. That OO is isolated in c is 7.1.10. That c is physically possible is 7.1.13. That odie is the only mobile object in OO follows from 7.1.9 and 1.8. That odie is motionless at the beginning of c is 7.1.14. That odie is convex follows from the fact that it spherical (7.1.1). That the top of the funnel points up follows from 7.2.5, and 7.1.7. That the top of the funnel has a fixed source follows

from 7.1.9. That the top of the funnel is in standard position in the startscene follows from 7.1.8. That the top of the funnel is above the starting place of the die follows from 7.1.15. Thus 7.2.4 applies, and the die cannot exit the top.

Lemma 7.2.7: The die does not ever abut the top of the funnel.

$$S \in \text{scenes}(c) \wedge \\ \text{abut}(\text{place}(\text{odie}, S), \text{place}(\text{ofunnel}, S), XXXA') \Rightarrow \\ \text{disjoint}(XXXA, \text{top}(\text{place}(\text{ofunnel}, S), \text{vup}))$$

Proof: The funnel and the die are everywhere smooth (1.9) and bounded (1.10). Therefore, the surface normal to the funnel at a top point is vertically up (7.1.7). Hence, if the die abuts the top, the surface normal to the die is vertically down. Since the die is convex, a downward surface normal only occurs at the bottom of the die. Hence, all of the die, including the centre-of-mass, is higher than the contact point. The remainder of the proof is as in lemma 7.2.4.

Lemma 7.2.8: Let XXS be a sphere. Let XXF be an s-tube generated from a planar figure with a convex inner boundary, with a radius greater than the radius of the sphere. If XXS abuts the interior surface of XXF , it does so in a single point.

$$[\text{sphere}(XXS) \wedge \text{s-tube}(XXT) \wedge \\ XXXT = \text{solid-of-revolution}(XXF, XXL) \wedge \\ \text{distance}(XXT, XXL) > \text{radius}(XXS) \wedge \\ \text{planar}(XXF \cup XXL) \wedge \\ \text{convex-side}(\text{inner-dside}(XXF, XXL), XXF) \wedge \\ \text{abut}(XXS, XXT, XXXA) \wedge \\ XXXA \subset \text{base}(\text{inner-boundary}(XXT))] \Rightarrow \\ \text{single-point}(XXXA)$$

Proof: Geometric theorem.

Lemma 7.2.9: If a convex shape abuts the inside of a p-tube, then any abutment points of the shape with the p-tube lies on the top, or the bottom, of the p-tube.

$$[\text{p-tube}(XXT, P1, P2, V) \wedge \text{convex}(XXO) \wedge \\ \text{abut}(XXO, \text{tube-inside}(XXT, P1, P2), XXXB) \wedge \\ \text{abut}(XXO, XXT, XXXA)] \Rightarrow \\ [XXXA \subseteq \text{top}(XXT, V) \vee XXXA \subseteq \text{bottom}(XXT, V)]$$

Proof: Geometric theorem.

Lemma 7.2.10: If any part of the object odie is in the inside of the funnel ofunnel, then odie is either disjoint from ofunnel or odie abuts ofunnel in a single point in the internal surface.

$$[S \in \text{scenes}(c) \wedge \text{not}(\text{disjoint}(\text{place}(\text{odie}, S), \\ \text{place}(\text{s-tube-inside}(\text{ofunnel}, S))))] \Rightarrow \\ [[\exists_x \text{abut}(\text{place}(\text{odie}, S), \text{place}(\text{ofunnel}, S), \{X\}) \wedge \\ X \in \text{place}(\text{inner-boundary}(\text{ofunnel}, S))] \vee \\ \text{disjoint}(\text{place}(\text{ofunnel}, S), \text{place}(\text{odie}, S))]$$

Proof: Geometrically, odie and ofunnel must either be disjoint, abut, or overlap; physically, they cannot overlap (4.3, 7.1.13). Since odie is convex (7.1.1), and ofunnel is a p-tube (7.2.5), if any part of odie is in the inside of ofunnel, then all abutment points of odie with ofunnel are in the inner boundary of ofunnel (7.2.9). But odie and ofunnel satisfy the conditions in (7.2.8), by virtue of 7.1.1, 7.2.5,

7.1.4, 7.1.5, and 7.1.6. Hence, there is only one abutment point between odie and the inner boundary of ofunnel, which proves the result.

Lemma 7.2.11: If odie abuts the ground, then it is disjoint from the inside of the funnel.

$$\text{abut}(\text{odie}, \text{oground}, XXXA) \Rightarrow \\ \text{disjoint}(\text{odie}, \text{s-tube-inside}(\text{ofunnel}))$$

Proof: If odie abuts the ground, then any point in odie is within diameter(odie) of the ground. Therefore, the height of any point in odie minus the height of any point in the ground is at most the diameter of odie. But all points in ofunnel are more than diameter(odie) above any point in the ground (7.1.11), and all points in the inside of ofunnel are above the bottom of ofunnel. Hence, the inside of ofunnel must be disjoint from odie.

Lemma 7.2.12: No point on the inner surface of a smooth s-tube constructed from a planar figure with a convex inner boundary points vertically upward.

$$[\text{s-tube}(XXT) \wedge XXXT = \text{solid-of-revolution}(XXF, XXL) \\ \text{planar}(XXF \cup XXL) \wedge \\ \text{convex-side}(\text{inner-side}(XXF, XXL), XXF) \wedge \\ \text{smooth}(XXT, X) \wedge X \in \text{inner-boundary}(XXT)] \Rightarrow \\ \text{surf-norm}(XXT, X) \neq \text{vup}$$

Proof: Geometric theorem.

Lemma 7.2.13: A convex mobile object which abuts only the bottoms of other objects is not possibly supported.

$$[\text{convex}(O) \wedge \text{mobile}(O) \wedge \\ \forall_{OP, XXXA} [\text{abut}(\text{place}(O, S), \text{place}(OP, S), XXXA) \Rightarrow \\ XXXA \subset \text{bottom}(OP, \text{vup})]] \Rightarrow \\ \text{not}(\text{possibly-supported}(O, S))$$

Proof: The surface normal to any of these other objects OP at their bottom is vertically down (1.9) hence, the surface normals to O there are all vertically up; hence, they do not have negative dot products with vup so that O is unsupported (3.4).

Lemma 7.2.14: In any stable scene, odie is disjoint from the inside of ofunnel.

$$[\text{possibly-stable}(S) \wedge S \in \text{scenes}(c)] \Rightarrow \\ \text{disjoint}(\text{place}(\text{odie}, S), \text{place}(\text{s-tube-inside}(\text{ofunnel}, S)))$$

Proof: If odie is not disjoint from the inside of ofunnel then it does not abut the ground (7.2.11); since it is isolated from everything but the ground and the funnel, it does not abut anything but the funnel (7.1.10, 7.1.2, 1.6). Furthermore, if it is not disjoint from the inside, then any abutment points are either on the inner boundary, or on the top, or on the bottom (7.2.9, 7.2.10). It cannot attain a position where it abuts the funnel on top (7.2.7). If it abuts on the inner boundary, then it is abutting in a single point (7.2.10), with a nonvertical surface normal (7.2.12); hence, the surface normal of the die cannot be vertically downward. The die, being spherical (7.1.1), can only be supported at a single point if its surface normal is downward at that point (3.5). Thus, it is not supported on the inner boundary. If it abuts on the bottom, then it is unsupported, by (7.2.13). If it does not abut at all, then it

is certainly unsupported (3.4). Since it is unsupported, the scene is unstable (3.3).

Theorem 7.2.15: The die comes out the bottom of the funnel.

exits(odie, s-tube-bottom(ofunnel), c)

Proof: Since c is infinite (7.1.12), it must end in some possibly stable state (7.1.13, 4.5, 4.6, 4.2). No stable state can occur until the die is disjoint from the inside of the funnel. (7.2.14). Therefore, the die must either exit the top or the bottom (7.2.1). Since it cannot exit the top (7.2.6) it must exit the bottom.

8. CONCLUSIONS

The strengths and limitations of this theory are evident. On the positive side: We have given a formal analysis of a class of problems far beyond the scope of any previous AI theory and we have completed the analysis within the boundaries of first-order deduction. Our analysis suggests that a qualitative physics for solid objects should include the following features, among others:

- A rich geometrical theory, including topological, metric, and differential descriptors, and special shapes.
- A formulation of physical rules in terms of behaviours over extended intervals of time. Examples of such rules include constraints; conservation laws, especially conservation of energy; and the principle that a physical system achieves a stable resting point. Similar rules have been used by Hayes³³ in his analysis of reasoning about liquids.
- The ability to determine the existence of a stable configuration of objects within qualitatively described geometrical constraints.
- The ability to calculate, exactly or qualitatively, important physical parameters such as the centre of mass. (Bundy and Byrd³⁴ present some interesting techniques for this kind of problem.)

It is also important to note what concepts have been unnecessary in solving these problems. For the examples in section 3, we did not have to reason about velocities, except the zero velocity, momentum, or angular momentum. If we may use the cited rules which determine possible stability directly from geometrical specifications, we may also omit any reasoning about forces or torques. This is not to suggest that these concepts can be ignored in a larger space of examples, nor that these examples could not be given an alternative analysis which did not use these concepts. Still, it is important to know how powerful a limited set of concepts is in solving particular classes of problems.

On the negative side: We have not shown that this type of analysis is extensible to cover all, or most, qualitative reasoning in this domain. We have not shown that such an extension would be, in the long run, any more parsimonious than simply enumerating special cases, as in the rule-based method rejected in section 2. We have not shown that any effective computational methods can be developed on the basis of this theory.

Three major open problems must be solved before a wide range of such problem can be solved robustly and automatically. First, we need a general geometric

description capable of representing arbitrary shapes and positions, and physically possible motions; of representing partial information of various kinds; and of supporting the efficient retrieval of all the geometric relations mentioned above. These are enormously strong requirements; no existing representation comes close to satisfying them, and we have made little progress on this problem in four years of work. Indeed, the difficulty of this problem has led Forbus³⁵ to claim that there cannot be any qualitative kinematics. Still, human beings do often make sensible physical predictions based on partial information; they must be representing this information somehow.

The second open problem is the analysis of extended collisions between objects. We need general principles that allow us to predict approximately the eventual state of two or more colliding objects without requiring us to determine every interaction between the two objects leading to that state. Such principles are very often needed in commonsense reasoning; for example, to predict that a rectangular chalk eraser dropped from six feet high onto the floor will end up at rest less than ten feet away. The only obvious general principles available are conservation of energy and, if neither object is fixed, conservation of momentum. These do not, in general, offer sufficient constraints to solve the problem.

The third problem, which is closely connected to the second, is the inclusion of metric time. We would like to be able to predict that the die will come out the funnel in a matter of seconds. Again, this will require a much more powerful analysis of object interactions. The practical solution to this problem and the previous one may well require the use of heuristics that are generally but not universally valid.

It may be that our difficulties are caused by sticking too closely to the known theory of mechanics. This theory is so powerful and so tractable in so many cases that the AI researcher is always tempted to turn to it for his analysis. It may be, however, that commonsense problems can be solved effectively only by using some substantially different set of concepts and rules. If so, a more productive avenue of research might be to study commonsense reasoning about physical systems which are very much harder to analyse mathematically: fluids^{33,36}, flexible solids, and the like. The difficulty of the precise analysis of such systems might be so overwhelming as to lead us no choice but to find a tractable commonsense level description.

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