Basic Algorithms, Assignment 6, SOLUTIONS

Due, Thursday, Oct 25

1. Consider the directed graph on $1, \ldots, n$ so that $\texttt{Adj}[i]$ consists of the points $i+1, \ldots, n$ in that order. Describe what happens when the depth first search algorithm is run where the vertices themselves are in order $1, \ldots, n$. Describe when the vertices turn grey and then black. Define `GOODORDER` (as in the webnotes) as a stack which has (at the end) the vertices in reverse order of when they turned black. What will be the order of the stack `GOODORDER`.
   Solution: $\texttt{DFS}[1]$ will call $\texttt{DFS}[2]$ which calls down the line to $\texttt{DFS}[n]$. So $1, 2, \ldots, n$ turn grey in that order. Then $\texttt{DFS}[n]$ finishes, n turns black, reverts to $\texttt{DFS}[n-1]$ so it finishes, $n-1$ turns black, this continues up the line to $\texttt{DFS}[1]$ which finishes and turns black. The stack `GOODORDER` is $1, 2, \ldots, n$.

2. Same question as above except that the vertices are in reverse order $n, \ldots, 1$.
   Solution: We first apply $\texttt{DFS}[n]$. n turns grey and then black. Then we apply $\texttt{DFS}[n-1]$, $n-1$ turns grey. As its neighbor n is already black it has nothing to do, $n-1$ turns black. We go up the chain until applyiing $\texttt{DFS}[1]$. 1 turns grey. As its neighbor 2 is already black it has nothing to do, 1 turns black. The stack `GOODORDER` is $1, 2, \ldots, n$. (Note that `GOODORDER` is the same in both instances. This has to be the case as we have a DAG with only one topological ordering and so `GOODORDER` must be it. However, the processes were quite different. For example (taking $n = 100$, 37 is white when 53 turns black in the second problem but 37 is grey when 53 turns black in the first problem.

3. Page 107, Exercise 1.
   Solution: a must be first and f last so lets ignore them. Now, effectively, we want to *merge* bc with de. There are *six* ways: $bcde, bdce$, $dbce, bdec$, $dbec, debc$. One way to count this is to note that the places where b, c are put in the list of four items determines the order, as they must be put in order and then d, e are in order in the other two places. There are $\binom{4}{2} = 6$ ways to choose two things out of four.

4. Consider the directed graph G on $1, 2, 3, 4, 5$ which has edges $(1, 2), (1, 3)$ (and no others). How many topological orders does it have?
   Solution: The values $1, 2, 3$ can be either in the orders 123 or 132. Given these, place 4 in one of four places 4123, 1423, 1243 ,1234;

4132, 1432, 1342, 1324. Then 5 fits into one of five places, e.g., 54123, 45123,41523,41253, 41235 for the first one. So each of the eight orderings of $1, 2, 3, 4$ become 5 orderings of $1, 2, 3, 4, 5$ giving forty permutations.

5. (**REVISED:** Below is the problem that I wanted to ask. Grading was based on the original problem!) Consider the graph on vertices $1, \ldots, 20$ with the following edges: $(i, i+1), 1 \leq i < 10$; $(10, 1)$; $(i, i+1), 11 \leq i \leq 20$; $(20, 11)$; $(5, 13)$. Draw a nice picture of this graph. What are its strong components. Draw the supergraph.

He rarely copied box scores into the Book, but today it seemed the right thing to do. All those zeroes! He decided for zeroes he'd use red ink. Zero: the absence of number, an incredible idea! Only infinity compared to it, and no batter could hit an infinite number of home runs - no, in a way, the pitchers had it better. Perfection was available to them.
The Universal Baseball Association, Inc., J. Henry Waugh, Prop.
– Robert Coover