Basic Algorithms, Assignment 1, Solutions

Due, Tuesday, Sept 11

Instructor: Prof. Joel Spencer
Office: wwh829
Email: `spencer@cs.nyu.edu`
Phone: x83219
Office Hours: Wednesday 9:30-11 or by appointment.
Course website:
`www.cs.nyu.edu/cs/faculty/spencer/basicalg/index.html`
or go to Prof. Spencer's home page (under CS faculty) and look for pointer.

1. Consider the Gale-Shapley algorithm with n men and n women. Suppose each man has a different woman first on his list. Describe how the algorithm will work. How many times will the WHILE loop be employed?
   Solution: Each man will propose to a free woman who will say yes, until all men (and hence all women) are matched. The WHILE loop will be employed n times.

2. With $n = 4$ select random permutations for each man and each woman as your data and then run the Gale-Shapley algorithm by hand, giving the intermediate positions until reaching the final matching.
   Solution:
   Man 1: 1 2 3 4
   Man 2: 1 3 2 4
   Man 3: 2 3 1 4
   Man 4: 4 3 2 1
   Women 1: 2 1 3 4
   Women 2: 2 1 3 4
   Women 3: 1 2 3 4
   Women 4: 4 1 3 4

   Step 1: Man 1 proposes to Women 1 who accepts.
   Step 2: Man 2 proposes to Women 1, who accepts. Man 1 is now free.
   Step 3: Man 1 proposes to Women 2, who accepts.
   Step 4: Man 3 proposes to women 2, who rejects staying with Man 1
   Step 5: Man 3 proposes to women 3, who accepts.
   Step 6: Man 4 proposes to women 4, who accepts.

The final results are as follows:
Man 1 marries Women 2
Man 2 marries Women 1
Man 3 marries Women 3
Man 4 marries Women 4

3. Construct an example with $n = 5$ in which man 1 proposes to all women $1, 2, 3, 4, 5$ and in which women $1, 2, 3, 4$ initially accept his proposal and later jilt him and in which woman 5 accepts his proposal and is his final mate.
Solution (one of many): The x's can be arbitrary.
Man 1: 12345
Man 2: 1xxxx
Man 3: 2xxxx
Man 4: 3xxxx
Man 5: 4xxxx
Woman 1: 21xxx
Woman 2: 32xxx
Woman 3: 43xxx
Woman 4: 54xxx
Woman 5: 1xxxx

Now if the proposals go in the "right" order here is what will happen. (a to b means Man a proposes to Woman b)
1 to 1: Accept
2 to 1: Accept, jilt M1
1 to 2: Accept
3 to 2: Accept, jilt M1
1 to 3: Accept
4 to 3: Accept, jilt M1
1 to 4: Accept
5 to 4: Accept, jilt M1
1 to 5: Accept

4. Page 22, Exercise 1
Solution: Very False! Indeed, the data may make this impossible. Maybe every man's first choice has him as her last choice!

5. Page 22, Exercise 2
Solution: True, as otherwise m, w would form an instability.

No one has yet programmed a computer to be of two minds about a hard problem, or to burst out laughing, but that may come.
– Lewis Thomas