1. Consider the Gale-Shapley algorithm with $n$ men and $n$ women. Suppose each man has a different woman first on his list. Describe how the algorithm will work. How many times will the WHILE loop be employed?
Solution: Each man will propose to a free woman who will say yes, until all men (and hence all women) are matched. The WHILE loop will be employed $n$ times.

2. With $n = 4$ select random permutations for each man and each woman as your data and then run the Gale-Shapley algorithm by hand, giving the intermediate positions until reaching the final matching.
Solution:
Man 1: 1 2 3 4  
Man 2: 1 3 2 4  
Man 3: 2 3 1 4  
Man 4: 4 3 2 1  
Women 1: 2 1 3 4  
Women 2: 2 1 3 4  
Women 3: 1 2 3 4  
Women 4: 4 1 3 4  

Step 1: Man 1 proposes to Women 1 who accepts.  
Step 2: Man 2 proposes to Women 1, who accepts. Man 1 is now free.  
Step 3: Man 1 proposes to Women 2, who accepts.  
Step 4: Man 3 proposes to women 2, who rejects staying with Man 1  
Step 5: Man 3 proposes to women 3, who accepts.  
Step 6: Man 4 proposes to women 4, who accepts.
The final results are as follows:
Man 1 marries Women 2
Man 2 marries Women 1
Man 3 marries Women 3
Man 4 marries Women 4

3. Construct an example with \( n = 5 \) in which man 1 proposes to all women 1, 2, 3, 4, 5 and in which women 1, 2, 3, 4 initially accept his proposal and later jilt him and in which woman 5 accepts his proposal and is his final mate.
Solution (one of many): The \( x \)'s can be arbitrary.
Man 1: 12345
Man 2: 1xxxx
Man 3: 2xxxx
Man 4: 3xxxx
Man 5: 4xxxx
Woman 1: 21xxx
Woman 2: 32xxx
Woman 3: 43xxx
Woman 4: 54xxx
Woman 5: 1xxxx

Now if the proposals go in the “right” order here is what will happen.
(a to b means Man a proposes to Woman b)
1 to 1: Accept
2 to 1: Accept, jilt M1
1 to 2: Accept
3 to 2: Accept, jilt M1
1 to 3: Accept
4 to 3: Accept, jilt M1
1 to 4: Accept
5 to 4: Accept, jilt M1
1 to 5: Accept

4. Page 22, Exercise 1
Solution: Very False! Indeed, the data may make this impossible.
Maybe every man’s first choice has him as her last choice!

5. Page 22, Exercise 2
Solution: True, as otherwise \( m, w \) would form an instability.
No one has yet programmed a computer to be of two minds about a hard problem, or to burst out laughing, but that may come.
– Lewis Thomas