Basic Algorithms, Assignment 9

Due, Thursday, Nov 15

1. Page 190, Exercise 5. For definiteness, let $x[i]$, $1 \leq i \leq n$ be the locations of the n houses (in miles from the western endpoint) and let $y[j]$, $1 \leq j \leq s$ be the placements of your stations. *Assume* the $x[i]$ are already ordered. Design your algorithm, give a cogent argument for its correctness, and analyze its time as a function of n.

2. Suppose we are given the Minimal Spanning Tree T of a graph G. Now we take an edge $\{x, y\}$ of G which is not in T and reduce its weight $w(x, y)$ to a new value w. Suppose the path from x to y in the Minimal Spanning Tree contains an edge whose weight is bigger than w. Prove that the old Minimal Spanning Tree is no longer the Minimal Spanning Tree.

3. Let $n = 2^t$. Consider the alphabet $S = \{1, \ldots, n\}$ with frequencies $f[i] = 2^{-i}$, $1 \leq i \leq n-1$ and $f[n] = 2^{-n+1}$. Describe how the Huffman Code Algorithm with work, the final code γ, and $ABL[\gamma]$, the Average Bits per Letter for the code. Let γ^* denote the code that sends i into the binary expansion of $i-1$, where each binary expansion is given t bits. What is $ABL[\gamma^*]$ as a function of n.

4. Suppose we ran Kruskal's algorithm on a graph G with n vertices and m edges, no two costs equal. Suppose the the $n-1$ edges of minimal cost form a tree T.

   (a) Argue that T will be the minimal cost tree.

   (b) How much time will Kruskal's Algorithm take. (Assume it stops when it finds the MST.)

   (c) We define Dumb Kruskal. It is Kruskal without the SIZE function. For $UNION[u, v]$ we follow u, v down to their roots x, y as with regular Kruskal but now, if $x \neq y$, we simply reset $\pi[y] = x$. We have the same assumptions on G as above. How long could dumb Kruskal take. Describe an example where it takes that long. (You can imagine that when the edge u, v is given an adversary puts them in the worst possible order to slow down your algorithm.)

People wish to learn to swim and at the same time to keep one foot on the ground.
– Marcel Proust