# Three Finger Optimal Planar Grasp

B. Mishra M. Teichmann*

**Abstract**
In this paper, we study various algorithmic questions regarding the computation of an optimal three finger planar grasp. We present a novel $O(n^2 \log n)$-time algorithm to compute such an optimal grasp for an arbitrary simple $n$-gon. We also discuss several variations on the problem and many intriguing open questions in the area that remain unsolved.

## 1. Introduction

In robotics and manufacturing, the problem of grasping, fixturing and workholding occupies a central position. While the question of analyzing and synthesizing *a* closure grasp or *a* fixture is fairly well-studied (see for instance [1, 5, 6, 7, 9, 10, 12, 16]), the question of formalizing a satisfactory notion of a grasp metric, and devising efficient algorithm for synthesizing grasps of *good quality* has received relatively less attention. A systematic exploration in this direction was initiated in the work of Kirkpatrick, Mishra and Yap [3], where a deep interconnection between the problem of optimal grasp synthesis and certain quantitative versions of Helly-type theorems in combinatorial geometry (namely, quantitative Steinitz's theorem) plays a key role. However, the resulting algorithmic questions in the most general setting appear to be intractable. For a through discussion of these issues, consult [8].

In order to better understand the underlying structure as well as to provide practical solutions in the simpler settings (as more common in manufacturing), we have directed our attention to the cases where we study lower-dimensional objects (2-D or $2\frac{1}{2}$-D) and of simpler geometry (polygonal objects) or simpler robot hands. In this paper, we explore this problem for two-dimensional polygonal objects with hands of relatively few fingers. Other works in a similar spirit include those of Ferrari and Canny [2], Li and Sastry [4], Brost and Goldberg [1] and Trinkle [15].

In this paper, we study various algorithmic questions regarding the computation of an optimal three finger planar grasp. We present a novel $O(n^2 \log n)$-time algorithm to compute such an optimal grasp for an arbitrary simple $n$-gon. We also discuss several variations on the problem and many intriguing open questions in the area that remain unsolved.

## 2. Preliminary

We wish to obtain the best three-finger grasp of a planar polygonal object assuming non-frictional contacts. Note that in this case, since it is not possible to guarantee that the resulting grasp will have the force/closure properties, we are willing to sacrifice the condition requiring torque-closure. In other words, we wish only to achieve a three-finger grasp such that the smallest external force such a grasp can resist is as large as possible.

More formally given a simple $n$-gon $P$, we wish to choose three distinct points $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ on the interior of the edge segments of $P$ such that the following properties hold:

1. The unit inner normals $\mathbf{n}(\mathbf{p}_1)$, $\mathbf{n}(\mathbf{p}_2)$ and $\mathbf{n}(\mathbf{p}_3)$ are concurrent.
2. The unit inner normals $\mathbf{n}(\mathbf{p}_1)$, $\mathbf{n}(\mathbf{p}_2)$ and $\mathbf{n}(\mathbf{p}_3)$ positively spans the two-dimensional force space, i.e.,
$$(\forall \mathbf{w} \in \mathbb{R}^2)\ (\exists f_i \geq 0, 1 \leq i \leq 3)\ \mathbf{w} = \sum_{i=1}^{3} f_i \mathbf{n}(\mathbf{p}_i).$$
3. The unit normals are "well-balanced" in the sense that
$$\min\Big\{ |\mathbf{w}| : \mathbf{w} \in \mathbb{R}^2, \\ (\exists f_i \geq 0, 1 \leq i \leq 3)\ \chi(f_1, f_2, f_3) = 1 \\ \mathbf{w} = \sum_{i=1}^{3} f_i \mathbf{n}(\mathbf{p}_i) \Big\},$$
is as large as possible (among all choices of $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$). Here, $\chi(f_1, f_2, f_3)$ denotes a finger force constraint condition on the magnitude of the forces applied at the points of contact. For instance,
$$\chi_{con} : f_i \geq 0 \text{ and } \sum f_i \leq 1,$$
or
$$\chi_{max} : f_i \geq 0 \text{ and } \max f_i \leq 1.$$

*Courant Institute, NYU, USA.

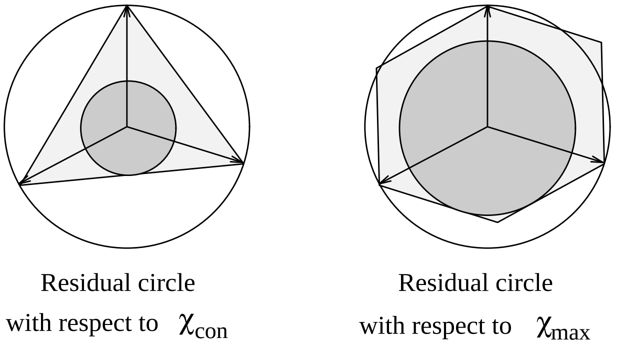


Figure 1: Grasp metrics associated with $\chi_{con}$ and $\chi_{max}$.

Thus the first property denotes the trivial torque equilibrium condition; the second property denotes the force closure condition and the third property measures the goodness of the grasp. In English, the third property says: under the condition $\chi_{con}$, we wish to maximize the radius of a disk, centered at origin and contained in the triangle formed by (convex hull of) the points (on the unit circle) corresponding to the vectors $\mathbf{n}(\mathbf{p}_1)$, $\mathbf{n}(\mathbf{p}_2)$ and $\mathbf{n}(\mathbf{p}_3)$. Similarly, under the condition $\chi_{max}$, we wish to maximize the radius of a disk, centered at origin and contained in the Minkowski sum of the points (on the unit circle) corresponding to the vectors $\mathbf{n}(\mathbf{p}_1)$, $\mathbf{n}(\mathbf{p}_2)$ and $\mathbf{n}(\mathbf{p}_3)$—a convex hexagon.

Let the corresponding radii be denoted as $\rho_{con}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ and $\rho_{max}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$, respectively. Note that, if the angle $\alpha_i$'s $(1 \leq i \leq 3)$ denote the angles between the inner normals then $\alpha_{max} = \max(\alpha_1, \alpha_2, \alpha_3) \geq 2\pi/3$ completely determines the radii

$$\begin{aligned} \rho_{con} &= \cos(\alpha_{max}/2), \quad \text{and} \\ \rho_{max} &= \sin \alpha_{max}. \end{aligned}$$

Thus both these metrics are monotonically decreasing functions of $2\pi/3 \leq \alpha_{max} \leq \pi$, and it suffices to minimize $\alpha_{max}$. However, for the sake of the ease of exposition, we will frequently use $\rho = \rho_{con}$, and refer to it as the "*residual radius*" of $\mathbf{n}(\mathbf{p}_1)$, $\mathbf{n}(\mathbf{p}_2)$ and $\mathbf{n}(\mathbf{p}_3)$. The optimal value of residual radius is denoted by $\rho^*$.

Note that given an edge $e = ab$ of the polygon $P$, for every point $p \in ab$, $\mathbf{n}(\mathbf{p})$ defines a unique point $q(e)$ on the unit circle in $\mathbb{R}^2$. Thus we may simply refer to this point on the unit circle by $q(e)$. Henceforth, let the edges of the $n$-gon be given as $E = \{e_1, e_2, \ldots, e_n\}$ and the corresponding points on the unit circle be $Q = \{q_1, q_2, \ldots, q_n\}$, where $q_i = q(e_i)$ $(1 \leq i \leq n)$.

## 3. A Cubic Time Algorithm

We may note at this point that there is a trivial $O(n^3)$ time algorithm to find an optimal grasp of a simple $n$-gon, $P$, by exhaustively enumerating all edge triples of $P$ and by examining each triple successively. In order for an edge triple $(e_i, e_j, e_k)$ to produce three necessary optimal contact points, it must be the case that $(q_i, q_j, q_k)$ form a triangle with a positive residual radius of $\rho^*$—a condition that can be checked easily in $O(1)$ time. However, this is not sufficient—since we must check that there are three points $\mathbf{p}_i \in e_i$, $\mathbf{p}_j \in e_j$ and $\mathbf{p}_k \in e_k$ satisfying the torque equilibrium condition; namely, that $\mathbf{n}(\mathbf{p}_1)$, $\mathbf{n}(\mathbf{p}_2)$ and $\mathbf{n}(\mathbf{p}_3)$ are concurrent meeting at some point $c$.

This is not hard but requires some thought. We proceed as follows: Consider an edge $ab$ of $P$. Let $HP(a, ab)$ be the open half plane containing $ab$ and delimited by a line containing $a$ and normal to $ab$ and similarly, let $HP(b, ab)$ be the open half plane containing $ab$ and delimited by a line containing $b$ and normal to $ab$. Let

$$\text{slab}(e) = HP(a, ab) \cap HP(b, ab),$$

where $e = ab$.

Then it is easy to see that for a triple of edges $(e_i, e_j, e_k)$ to satisfy the torque equilibrium condition, it is necessary and sufficient that

$$\text{slab}(e_i) \cap \text{slab}(e_j) \cap \text{slab}(e_k) = C \neq \emptyset.$$

The point of concurrency $c \in S$, and the contact points $\mathbf{p}_i$, $\mathbf{p}_j$ and $\mathbf{p}_k$ are determined by the normals from $c$ onto the edges $e_i$, $e_j$ and $e_k$.

Thus our previous arguments can be summarized to be saying that an edge triple $(e_i, e_j, e_k)$ defines an optimal grasp if $\text{slab}(e_i) \cap \text{slab}(e_j) \cap \text{slab}(e_k)$ is nonempty and that the triangle formed by the corresponding points on the unit circle has a positive residual radius of $\rho^*$, maximal among all choices of edge triples. These considerations yield an $O(n^3)$-time algorithm.

## 4. An Improved Subcubic Algorithm

Next, we ask if it is possible to improve upon the trivial $O(n^3)$-time algorithm. Here, we present an $O(n^2 \log n)$-time algorithm for finding the optimal three fingered planar grasp for an arbitrary simple polygon and some simple improvements for convex polygons.

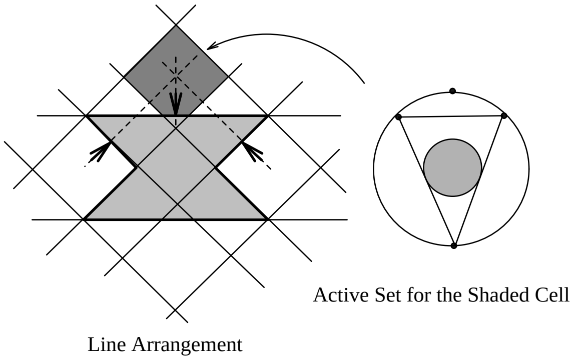


Figure 2: The line arrangement associated with an object.

We first describe the algorithm assuming that the polygon $P$ is nondegenerate (in the sense that will be made precise later) and then remark on how the nondegeneracy can be eliminated by a simple modification to the algorithm.

The algorithm can be described as follows: First we create the two-dimensional line arrangement formed by a collection of lines consisting of three lines per edge, where the triplet of lines associated with an edge $ab$ are: (1) the line containing the edge $ab$, (2) the line normal to $ab$, containing $a$ and (3) the line normal to $ab$, containing $b$. Now consider a nonempty cell $C$ of this arrangement: we say a point $q = q(e)$ on the unit circle is *active* for this cell, if $\text{slab}(e) \supseteq C$. The subset of points on the unit circle (among the points $q_1, q_2, \ldots, q_n$ of $Q$) that are active for this cell $C$, is called its active set and denoted by $\text{active}(C) \subseteq Q$. Now, if we find three points $q_i$, $q_j$ and $q_k \in \text{active}(C)$, whose residual radius $\rho(C)$ is as large as possible (and positive), then it is seen that $\rho^*$ is simply the maximum of all $\rho(C)$'s taken over all cells of the arrangement.

Note that there are at most $O(n^2)$ cells altogether and as we go from one cell $C$ to its adjacent cell $C'$ then the $\text{active}(C')$ can be computed from the $\text{active}(C)$ by adding or deleting a point on the unit circle, depending on the line containing the $C \cap C'$. Of course, here we have tacitly assumed that the polygon is nondegenerate, in the sense that the all the lines on the arrangement are distinct, since otherwise $C \cap C'$ may belong to more than one line of the arrangement and thus require addition and deletion of more than one point of the set $Q$. Clearly, the active sets for all the cells can be computed in $O(n^2)$ time by visiting the cells of the arrangement, starting from a cell with an empty active set (such a cell exists sufficiently far away from the polygon $P$). However, computing the $\rho(C)$ for each cell may still take $O(n)$ time, thus forcing the entire procedure to take $O(n^3)$ time.

We circumvent this problem by the following simple trick: First of all we maintain the $\text{active}(C)$'s in a clockwise order in a dynamic balanced binary search tree. Since each update operation on this data structure takes $O(\log n)$ time, this increases the complexity of computing the active sets of all the cells to $O(n^2 \log n)$-time.

At any instant, we only remember $\tilde{\rho}$—the maximal residual radius seen so far. That is, $\tilde{\rho}$ is simply the maximum of those $\rho(C)$'s corresponding to only those cells $C$ that have been visited so far. We also remember the edge triple associated with the radius value $\tilde{\rho}$. When we go from a visited cell $C$ to an adjacent unvisited cell $C'$, we do one of two things: If going to the next cell entails deletion of a point, $q_i$, on the unit circle, then we only have to update the $\text{active}(C')$; the maximal residual radius of $C'$ cannot be larger than that of $C$ and thus $\tilde{\rho}$ remains unchanged. If going to the next cell, on the other hand, entails addition of a point, $q_i$ on the unit circle, then we have to both update the $\text{active}(C')$ and check if $\tilde{\rho}$ can be improved. If the maximal residual radius of $C'$, $\rho(C') > \tilde{\rho}$, then the associated triplet from $\text{active}(C')$ must involve the new point $q_i$ and two of the old points. How can we do this operation quickly?

First note that residual radii cannot take all possible values but only one of $\binom{n}{2}$ values, each value being determined by a pair of distinct points $q_l$ and $q_m$ and is equal to the radius of the circle that is centered at the origin and has the line containing $q_l$ and $q_m$ as tangent. All these radii can be sorted in $O(n^2 \log n)$ time and are denoted by

$$0 \leq \rho_1 \leq \rho_2 \leq \cdots \leq \rho_i \leq \cdots < 1$$

Suppose before visiting the cell $C'$ the maximal residual radius seen so far is $\tilde{\rho} = \rho_i$. When we go to the cell $C'$ (which requires adding the point $q_i$), we will successively test if it has a residual radius no smaller than $\rho_{i+1}$, $\rho_{i+2}$, etc. until we fail for some value $\rho_j$ $(j > i)$. Each such test can be performed in $O(\log n)$ time as explained below.

Let $i < k \leq j$, and we wish to test if $\text{active}(C')$ has three points involving $q_i$ and of residual radius $\geq \rho_k$. Consider a circle $C(\rho_k)$ of radius $\rho_k$ and centered at the origin. Two distinct points of $\text{active}(C')$ are said to be *mutually visible* if the line segment connecting these two points do not intersect the interior of $C(\rho_k)$. Thus our test succeeds if we can find a pair of mutually visible distinct points among the $\text{active}(C')$, each of which is also mutually visible with $q_i$. Let the *leftmost partner* of $q_i$ be the *last* mutually visible point of $q_i$ encountered, visiting the points of $\text{active}(C')$ in clockwise order starting from $q_i$. We call this point $LP(q_i)$. Similarly, we define the the *rightmost partner* of $q_i$ by visiting the points of $\text{active}(C')$ in anti-clockwise order, and call it

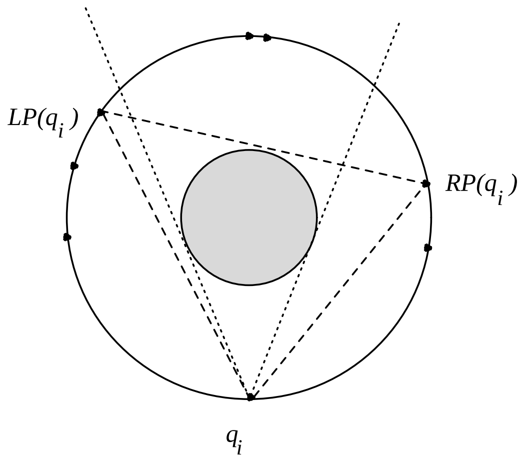


Figure 3: Test involving $q_i$ and a possible residual radius value of $\rho_k$.

$RP(q_i)$. Since the active points of $C'$ are kept in their sorted order in a balanced search structure, both $LP(q_i)$ and $RP(q_i)$ can be computed in $O(\log n)$ time. Then it only remains to check that $LP(q_i)$ and $RP(q_i)$ are mutually visible, a step that can be accomplished in $O(1)$ time.

Thus, we can keep track of $\tilde{\rho}$ by performing a sequence of tests per each new cell, each of which takes $O(\log n)$ time. Note that while there is no a priori bound on the number of tests we may need to perform for a new cell, it should be obvious that all but the last test succeeds and the last test fails. Thus there are at most one test per cell that fails, and the totality of all such failed tests incur a cost of $O(n^2 \log n)$. On the other hand, if we have a successful test involving a radius value $\rho_k$, then *we shall never perform another successful test involving $\rho_k$, subsequently.* Thus, the total number of successful tests are bounded by the number possible radii values ($\binom{n}{2}$ of those) and altogether they incur a cost of $O(n^2 \log n)$. Clearly, when we are done visiting all the cells, we have the global maximal residual radius $\rho^*$ together with the edge triple, which readily give the three contact points, and we have spent $O(n^2 \log n)$ time.

If the polygon $P$ is degenerate then the resulting arrangement may force us to add and delete many points of $Q$ while going from a cell to its adjacent cell. If we enforce the discipline that all the deletions are performed before all the additions and each update is performed sequentially then the correctness of the algorithm still holds and the performance analysis goes through *mutatis utandis*. In summary, we have

**Theorem 4.1** *Given an arbitrary simple $n$-gon $P$, we can compute a three finger optimal grasp of $P$ in $O(n^2 \log n)$ time.* □

## 5. Improvements and Open Questions

There are several open questions related to the problem of finding optimal planar grasps. We briefly discuss these problems.

(1) Consider a variation on the above problem: Suppose we are given a simple polygon $P$ with certain subset of $\partial P$ designated as "forbidden" and its complement, "feasible." Assume that the feasible parts of the polygon consists of at most $K$ segments (the edge segment $ab$ being allowed to be a point $a$ ($a = b$), in the degenerate case). We are asked to find an optimal three-finger grasp of the polygon with none of the fingers on a forbidden region. Using a small variation of the above algorithm, we can solve this problem in $O(k^2 \log k)$ time—only modify the line arrangement to consist of the following triple of lines per feasible edge segment $ab \subset e$, where $e$ is an edge of $P$: (1) the line containing $e$, (2) the line normal to $e$ and containing $a$, and (3) the line normal to $e$ and containing $b$. If the edge segment is a point $a \in e$ then the above situation degenerates to two lines, one containing $e$ and the other normal $e$ at $a$.

(2) We do not know whether there is a better solution for the above problem with improved complexity. For instance, it is not even clear whether there are $O(n)$ time algorithms for objects with simpler geometry, e.g., convex objects. We have an $O(n)$ solution only for what we shall refer to as *circular convex* polygonal objects. A convex polygon $P$ will be called *circular* if there is a point $c$ in its interior (its *center*) so that the line segment from $c$ to a line containing an edge $e$ and normal to $e$ is entirely within $P$. For instance, the convex hull of a set of points on a circle defines a circular polygon (thus, the name). Note that in this case,

$$\bigcap_{e \in E} \mathrm{slab}(e) \neq \emptyset,$$

where $E$ is the set of edges of $P$. Clearly, we can find a small neighborhood $U$ of the center $c$ such that active($U$) is all of $Q$. In this case, the problem reduces to simply finding three points $q_i$, $q_j$ and $q_k \in Q$ such that the residual radius of the resulting triangle is as large as possible.

We need to extend the notion of residual radius as follows: The residual radius of a triangle $\Delta$ is the *signed* radius of the largest disk centered at the origin that is either fully outside or fully inside $\Delta$, the sign being positive or negative depending on whether the disk is inside or outside $\Delta$ respectively.

Assume that the points of $Q$ are ordered in the anticlockwise order as

$$q_1 > q_2 > \cdots > q_n.$$

for any point $q \in Q$ its successor, $\text{succ}(q)$, is the point immediately following it in the clockwise order.

We start with three arbitrarily chosen distinct points, say $u_0 = q_1$, $u_1 = q_2$ and $u_2 = q_3$, for instance. At any instance, assume that, we have three points $u_0$, $u_1$ and $u_2$, at least two of which are distinct, and

$$u_0 \geq u_1 \geq u_2 \geq u_0$$

There are two cases to consider: (1) they are not all distinct, i.e., $u_i = u_{i+1}$ and (2) they are all distinct and the residual disk touches the edge $u_{i-1}u_i$.

In the first case, we advance the "forward point" $u_i$ (i.e, replace $u_i$ by $\text{succ}(u_i)$) with the hope of making the points distinct (this may not succeed and lead to further advancements of this kind). In the second case, we advance the "backward point $u_i$ (i.e, replace $u_i$ by $\text{succ}(u_i)$) with the hope of releasing the "limiting edge" $u_{i-1}u_i$ and thus possibly (but not always) increasing the residual radius.

The algorithm keeps advancing the forward or the backward point (as the case may be) while recording the maximal residual radius seen so far until $u_0$ returns to its initial position, at which point it halts and outputs the edge triple corresponding to the maximal residual radius. Since the polygon is a circular convex polygon, one can easily determine the contact points by taking the normals from the center of the polygon to each edge of the edge triple. The correctness and the complexity analysis of the algorithm can be shown in a manner similar to the discussions in section 5 of the paper by Kirkpatrick et. al. [3] and is omitted here. Note, however, that the above technique fails for arbitrary convex polygons if we relax the condition of circularity.

Note that the above technique can be easily adapted to the following problem: Given a simple polygon $P$ and a center $c \in \mathbb{R}^2$, find a 3-finger optimal grasp of $P$ such that the inner normals at the contact points go through $c$. This problem is solved by simply running the above algorithm starting with an active set, $\text{active}(U)$ of a small open neighborhood of $c$. The resulting algorithm takes $O(n)$ time.

(3) Sometimes, we wish to determine not just one optimal three finger grasp but all of them. Then we may use any one of this class of optimal grasps, depending on the task at hand. Clearly, the brute force $O(n^3)$ time algorithm will succeed to do so. Note that the algorithm of the previous section cannot be easily modified into a two pass algorithm, since addition of a new point (in the process of going from one cell to an adjacent cell) may create an $O(n)$ edge triplets of residual radius $\rho^*$. Here, we describe an $O(n^2 \log n)$ algorithm for the special case when the object is convex.

Let $P$ be a convex $n$-gon and let the possible residual radii (as in the preceding subsection) be given as

$$0 \leq \rho_1 \leq \rho_2 \leq \cdots \leq \rho_i \leq \cdots < 1.$$

We shall find the optimal residual radius $\rho^*$ by performing a binary search on the sequence of possible residual radii. For a given value of $\rho_i$, we can enumerate all the edge triples that lead to a residual radius of $\rho_i$ in $O(n^2)$ as follows: Corresponding to the possible radius value $\rho_i$, there are at most $O(n)$ edge pairs $(e_i, e_j)$'s such that the corresponding points $q_i$ and $q_j \in Q$ on the unit circle satisfy the property that the line determined by $q_i q_j$ is tangent to a circle $C(\rho_i)$ centered at the origin and of radius $\rho_i$. Now for each such edge pair, we need to check in $O(n)$ time if there is another edge $e_k$ such that $q_k \in Q \setminus \{q_i, q_j\}$ is mutually visible (with respect to $C(\rho_i)$) to both $q_i$ and $q_j$ and that

$$\text{slab}(e_i) \cap \text{slab}(e_j) \cap \text{slab}(e_k) \neq \emptyset.$$

We can thus enumerate all the $e_k$'s that succeed this test. The binary search only considers $O(\log n)$ different values of $\rho_i$'s and terminates with success with the largest possible value $\rho^*$ and enumerating all edge triples corresponding to $\rho^*$. It is then trivial to describe all possible three finger optimal planar grasps. Thus the algorithm has a time complexity of $O(n^2 \log n)$.

However, the algorithm applied to a nonconvex polygon leads to an $O(n^3)$-time algorithm, as in a pathological case, there may be $O(n^2)$ edge pairs to be considered for a given value of $\rho_i$. It is noteworthy that this algorithm is rather simple to implement and may perform well in practice. For instance, if one performs binary search on the real interval $[0, 1]$ (instead of the possible radii values), then for a random polygon this algorithm can compute in $O(n \log n \log(1/\epsilon))$ all three finger grasps whose corresponding residual radii lie in the range $[\rho, \rho^*]$ of size $< \epsilon$, for sufficiently small positive $\epsilon$.

(4) We still do not know how to find optimal $m$-finger planar grasp ($m \geq 4$) in time better than what can be obtained by the brute force algorithm taking time $O(n^{O(m)})$. For instance, it is not even clear if there is an algorithm to compute such an optimal grasp in time $O(n^{m-1} \text{polylog } n)$. The complication arises by virtue of the torque components that one has to consider in the case when $m \geq 4$.

Some progress has been made, by modifying the problem to that of choosing an optimal set of $m$-finger contact points out of a preselected $O(n)$ points on the boundary of the polygon, $\partial P$. For instance, we have an $O(n^3 \log n)$ time algorithm to find such an optimal four finger grasp in this case. The technique employed for this case is a generalization of the preceding algorithm involving binary search. We suspect that the algorithm generalizes to $m$ fingers ($m > 4$) and has a time complexity of $O(n^{m-1} \log n)$.

(5) In case of parallel jaw grippers and three-jaw grippers grasping an $n$-gon, one can compute the optimal grasps in time $O(n)$. The algorithms in these cases involve simply going around the object and trying all possible grasps [2]. It is not clear, if these grippers are comparable to multi-fingered hands in terms of how well they optimize various grasp metrics.

(6) Another problem of interest is to study the similar optimality problem in the case of "fixturing," where a polygonal object has to be fixtured by a set of toe-clamps that can be placed only at places designated by a set of toe-slots (which are usually arranged on a regular square grid) [1, 6, 7, 16]. The added difficulty arises because of the geometric constraints imposed by the toe-slots. It is easily seen that for a rectilinear object the optimal fixel (fixture element) placement can be determined in $O(n)$ time. However, the problem seems quite difficult even when we consider a convex polygonal object.

(7) Recently, we have been able to design "reactive hands" for grasping. These algorithms operate by determining a sensor-dependent binary vector and then actuating a small set of actuators by a simple table-lookup procedure[13, 14]. It remains an intriguing open question whether it is possible to design general multi-fingered reactive hands that always find an optimal grasp.

# References


[1] R.C. Brost and K.Y. Goldberg. "A Complete Algorithm for Synthesizing Modular Fixtures for Polygonal Parts," Submitted to the *1994 IEEE International Conference on Robotics and Automation*, 1994.

[2] C. Ferrari and J. Canny. "Planning Optimal Grasps," *Proceedings of the 1992 IEEE International Conference on Robotics and Automation*, Nice (France), pp. 2290–2295, 1992.

[3] D. Kirkpatrick, B. Mishra and C.-K. Yap. "Quantitative Steinitz's Theorem with Applications to Multifingered Grasping," *Discrete & Computational Geometry*, Springer-Verlag, New York, pp. 295–318,Volume **7**, Number 3, 1992.

[4] Z-X. Li and S. Sastry. "Task Oriented Optimal Grasping by Multifingered Robot Hands," *Proceedings of the 1987 IEEE International Conference on Robotics and Automation*, pp. 389–394, 1987.

[5] X. Markenscoff, L. Ni and C.H. Papadimitriou. "The Geometry of Grasping," *International Journal of Robotics Research*, pp. 61–74, Volume **9**, Number 1, 1990.

[6] B. Mishra. "Workholding—Analysis and Planning," *Proceedings: IEEE/RSJ International Workshop on Intelligent Robots and Systems*: IROS'91, pp. 53–57, Vol. **1**, Osaka, Japan, November 3–5, 1991.

[7] B. Mishra. "An Algorithmic Approach to Fixturing," *1994 NSF Design and Manufacturing Grantees Conference*, M.I.T., Massachusetts, Cambridge, January, 1994 (To appear).

[8] B. Mishra. "Grasp Metrics: Optimality and Complexity," *Proceedings of the Workshop on the Algorithmic Foundations of Robotics*: WAFR'94, Volume I, San Francisco, February 17–19, 1994.

[9] B. Mishra, J.T. Schwartz and M. Sharir, "On the Existence and Synthesis of Multifinger Positive Grips," Special Issue: Robotics, *Algorithmica*, Springer-International, pp. 541–558, Volume **2**, Number 4, 1987.

[10] B. Mishra and N. Silver. "Some Discussion of Static Gripping and Its Stability," *IEEE Transactions on Systems, Man and Cybernetics*, pp. 783–796, Volume **19**, Number 4, July/August, 1989.

[11] B. Mishra and M. Teichmann. "On Immobility," Special Issue: Robot Kinematics, *Laboratory Robotics and Automation:* International Journal, VCH Publisher, pp. 145–153, Volume **4**, 1992.

[12] V. Nguyen. "Constructing Force-Closure Grasps," *Proceedings of the 1986 IEEE International Conference on Robotics and Automation*, San Francisco, California (USA), pp. 1368–1373, 1986.

[13] M. Teichmann and B. Mishra. "Reactive Algorithms for Grasping Using a Modified Parallel Jaw Gripper," Submitted to the *1994 IEEE International Conference on Robotics and Automation*: ICRA'94, San Diego, California, 1994.

[14] M. Teichmann and B. Mishra. "Reactive Algorithms for 2 and 3 Finger Grasping," Submitted to the *1994 IEEE/RSJ International Workshop on Intelligent Robots and Systems*: IROS'94, Grenoble, (France), 1994.

[15] J.C. Trinkle. "A Quantitative Test for Form Closure Grasps," Department of Computer Science, Texas A&M University, College Station, TX, 1992.

[16] Y. Zhuang, K.Y. Goldberg and Y-C. Wong. "On the Existence of Modular Fixtures," Submitted to the *1994 IEEE International Conference on Robotics and Automation*, 1994.



APPEARS IN *Proceedings of the 1994 International Workshop on Intelligent Robots and Systems*: IRS'94, GRENOBLE, (FRANCE), JULY 11–14, 1994.