Due by July 14. Collaboration is allowed; please mention your collaborators.

Problem 1
Find $f(1), f(2), f(3), f(4)$ and $f(5)$ if $f(n)$ is defined recursively by $f(0) = 3$ and for $n = 0, 1, 2, \ldots$
(a) $f(n + 1) = -2f(n)$.
(b) $f(n + 1) = 3f(n) + 7$.
(c) $f(n + 1) = f(n)^2 - 2f(n) - 2$.
(d) $f(n + 1) = 3^{f(n)/3}$.

Problem 2
Give a recursive definition of the sequence $\{a_n\}, n = 1, 2, 3, \ldots$ if
(a) $a_n = 4n - 2$.
(b) $a_n = 1 + (-1)^n$.
(c) $a_n = n(n + 1)$.
(d) $a_n = n^2$.

Problem 3
Give a recursive definition of
(a) the set of odd positive integers.
(b) the set of positive integer powers of 3.
(c) the set of polynomials with integer coefficients.

Problem 4
Let $S$ be the subset of the set of ordered pairs of integers defined recursively by
Basis step: $(0, 0) \in S$.
Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.
List the elements of $S$ produced by the first five applications of the recursive step.

Problem 5
Determine whether each of these functions is $O(x)$ or not. If they are, provide the necessary constants and show that they work. If they aren’t, show that there are no such constants.
(a) $f(x) = 17x + 11$.
(b) $f(x) = x^2 + 1000$.
(c) $f(x) = x \log x$.
(d) $f(x) = x^4$.

Problem 6
Show that $f(x) = x^5 + 3x^3 + 6x^2 + 14$ is $\Theta(x^5)$. 
Problem 7
Determine whether $x^3$ is $O(g(x))$ when $g(x)$ is one of the following:
(a) $g(x) = x^2$.
(b) $g(x) = x^3$.
(c) $g(x) = x^2 + x^3$.
(d) $g(x) = x^2 + x^4$.
(e) $g(x) = \frac{x^3}{2}$.

Problem 8
Explain what it means for a function to be $O(1)$, $\Omega(1)$ and $\Theta(1)$.

Problem 9
Give a big-$O$ estimate for both of these functions. If you estimate that $f(x)$ is $O(g(x))$, use a simple function $g(x)$ of smallest order.
(a) $f(x) = (\log 5n^n + n^2)(n^4 + \log n^{24})$.
(b) $f(x) = (n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)$. 