Q1. Transporting people

You have to manage the transportation of a given number of people from a source to a destination in 3 days. The actual travel happens in buses during the daytime only. During night everybody has to be accommodated in some hotel. So a person travels in the following sequence, starting journey at the source in the morning, bus in day 1, hotel in night 1, bus in day 2, hotel in night 2 and then bus in day 3 and reaching destination in the evening.

Assume that all the hotels are at some or the other bus stop. Assume that during a single day the journey of any person uses a single bus. Each bus and hotel has capacity constraints. Number of buses servicing between a pair of stop is given to you. Assume that nobody else is using the buses and the hotels.

Determine whether such a transportation is possible or not.

Q2. Traveling by bus, tram and train

The transits points (stops) of a city are connected by transportation system. Between any pair of transit point there is either no direct service or they are connected by bus, tram or (inclusive) train service.

Given a starting transit point, determine where one can reach by using bus(es) first, then tram(s) and then train(s).

Q3. Transmitting a sequence of 0 and 1’s

You are given a sequence of 0 and 1’s to transmit. The transmitter has 4 buttons. Two buttons send 0 and 1. The other two buttons are shortcuts for sending 001 and 101 respectively. Determine the smallest number of button presses.

Q4. Arbitrage (Problem from CLRS, problem number omitted deliberately)

Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 46.4 Indian rupees, 1 Indian rupee buys 2.5 Japanese yen, and 1 Japanese yen buys 0.0091 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S. dollar and buy $46.4 \times 2.5 \times 0.0091 = 1.0556$ U.S. dollars, thus turning a profit of 5.56 percent.

Suppose that we are given n currencies $c[1], c[2], \ldots, c[n]$ and an $n \times n$ table $R$ of exchange rates, such that one unit of currency $c[i]$ buys $R[i, j]$ units of currency $c[j]$.

Give an efficient algorithm to determine whether or not there exists a sequence of currencies $c[i[1]], c[i[2]] \ldots c[i[k]]$ such that

$R[i[1], i[2]] \times R[i[2], i[3]] \times R[i[k-1], i[k]] \times R[i[k], i[1]] > 1$.

Analyze the running time of your algorithm.
Q5. Politics

Suppose you are the prime minister of a country. You are trying to select a subset of your ministers satisfying the following constraints. Each pair of ministers in your selected subset must be mutual friends. For each minister you know the number votes they have got. Each pair of minister may are mutual friends, enemies or (exclusive) neutral. Your spies are efficient enough to get these information for you. Your goal is to maximize the total number of votes in your selected subset.

Convince us that this problem is unlikely to be solved in polynomial (in the number of ministers) time by reducing a known NP-hard problem to it. Do give an exponential solution.