Fundamental Algorithms, Assignment 4
Due at: Monday, 6.00 PM, July 27, 2009

Instructor: Abhijit Guria

Q1. CLRS: Exercises 14.3-6 (MIN-GAP)
Show how to maintain a dynamic set Q of numbers that supports the operation MIN-GAP, which gives the magnitude of the difference of the two closest numbers in Q. For example, if Q = {1, 5, 9, 15, 18, 22}, then MIN-GAP(Q) returns 18 − 15 = 3, since 15 and 18 are the two closest numbers in Q. Make the operations INSERT, DELETE, SEARCH, and MIN-GAP as efficient as possible, and analyze their running times.
Hint: For every node, consider its predecessor.

Q2. Price fixing by a dictator
In a strange country, a dictator fixes price by his will. There are n types of commodities in that market. The dictator specifies a list containing pairs of commodities so that the unit price of the first commodity in a pair should be higher than that if the second commodity of the pair. E.g., if he wants unit price of apples to be higher than that of oranges he puts (apples, oranges) in that list. Give an algorithm which takes such a list and decides whether each commodity can be assigned some price satisfying the dictator.
When such an assignment is possible give an algorithm to find one such price which minimizes the sum of unit prices of the commodities. Assume that each unit price has to be some integral (positive!) multiple of unit amount of money.

Q3. Switching between public and private transport
A traveler wants to go from a source to a destination in a network of cities. Between every two cities she can either choose public transport or private one. She knows the costs of both of them. She wants to minimize the sum of total costs in dollars plus the number of times she switches from public transport to a private one.
Help her!

Q4. CLRS: Exercises 16.2-4
Professor Midas drives an automobile from Newark to Reno along Interstate 80. His car’s gas tank, when full, holds enough gas to travel n miles, and his map gives the distances between gas stations on his route. The professor wishes to make as few gas stops as possible along the way. Give an efficient method by which Professor Midas can determine at which gas stations he should stop, and prove that your strategy yields an optimal solution.

Q5. CLRS: Exercises 16.2-5
Describe an efficient algorithm that, given a set \{x_1, x_2, \ldots, x_n\} of points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Argue that your algorithm is correct.