Programming Languages

ML

G22.2110
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ML overview

- originally developed for use in writing theorem provers
- functional: functions are first-class values
- garbage collection
- strict
- strong and static typing; powerful type system
  - parametric polymorphism (somewhat like Ada generics)
  - structural equivalence
  - all with type inference!
- advanced module system
- exceptions
- miscellaneous features:
  - datatypes (merge of enumerated literals and variant records)
  - pattern matching
  - references (like “const pointers”)
- val k = 5;
val k = 5 : int

- k * k * k;
val it = 125 : int

‘it’ denotes the last computation

- [1, 2, 3];
val it = [1,2,3] : int list

- ["hello", "world"];
val it = ["hello","world"] : string list

- 1 :: [ 2, 3 ];
val it = [1,2,3] : int list
Operations on lists

- `null [1, 2];`  
  \> \textit{val it = false : bool}

- `null [ ];`  
  \> \textit{val it = true : bool}

- `hd [1, 2, 3];`  
  \> \textit{val it = 1 : int}

- `tl [1, 2, 3];`  
  \> \textit{val it = [ 2, 3 ] : int list}

- `[ ];`  
  \> \textit{val it = [ ] : 'a list}  
  \hspace{1cm} this list is polymorphic
A function *declaration*:

- `fun abs x = if x >= 0.0 then x else -x`
  `val abs = fn : real -> real`

A function *expression*:

- `fn x => if x >= 0.0 then x else -x`
  `val it = fn : real -> real`
Functions, II

- fun length xs =  
  if null xs  
  then 0  
  else 1 + length (tl xs);

val length = fn : 'a list -> int  

'a denotes a type variable; length can be applied to lists of any element type

The same function, written in pattern-matching style:

- fun length [] = 0  
  | length (x::xs) = 1 + length xs

val length = fn : 'a list -> int
Advantages of type inference and polymorphism:

- frees you from having to write types. A type can be more complex than the expression whose type it is, e.g., `flip`
- with type inference, you get polymorphism for free:
  - no need to specify that a function is polymorphic
  - no need to "instantiate" a polymorphic function when it is applied
Multiple arguments?

- All functions in ML take exactly one argument
- If a function needs multiple arguments, we can

1. pass a tuple:
   - `(53, "hello"); (*a tuple *)
   ```
   val it = (53, "hello") : int * string
   ```
   We can also use tuples to return multiple results.

2. use currying (named after Haskell Curry, a logician)
Another function; takes two lists and yields their concatenation

- fun append1 ([ ], ys) = ys
  
  | append1 (x::xs, ys) = x :: append1 (xs, ys);
val append1 = fn : 'a list * 'a list -> 'a list

- append1 ([1,2,3], [8,9]);
val it = [1,2,3,8,9] : int list
The same function, written in curried style:

- `fun append2 [] ys = ys
  | append2 (x::xs) ys = x :: (append2 xs ys);
val append2 = fn: 'a list -> 'a list -> 'a list

- append2 [1,2,3] [8,9];
val it = [1,2,3,8,9] : int list

- val app123 = append2 [1,2,3];
val app123 = fn : int list -> int list

- app123 [8,9];
val it = [1,2,3,8,9] : int list
But what if we want to provide the other argument instead, i.e. append \([8,9]\) to its argument?

- here is one way: (the Ada/C/C++/Java way)
  
  \[
  \text{fun appTo89 } \text{x}s = \text{append2 } \text{x}s \ [8,9] \]

- here is another: (using a higher-order function)
  
  \[
  \text{val appTo89 } = \text{flip append2 } \ [8,9] \]

\text{flip} is a function which takes a curried function and “flips” its two arguments. We define it on the next slide...
fun flip f y x = f x y

The type of `flip` is \((\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \beta \rightarrow \alpha \rightarrow \gamma\). Why?

- Consider \((f \ x)\). \(f\) is a function; its argument has the same type as \(x\).
  \[
  f : A \rightarrow B \quad x : A \quad (f \ x) : B
  \]

- Now consider \((f \ x \ y)\). Because function application is left-associative, \(f \ x \ y \equiv (f \ x) \ y\). Therefore, \((f \ x)\) must be a function, and its argument must have the same type as \(y\):
  \[
  (f \ x) : C \rightarrow D \quad y : C \quad (f \ x \ y) : D
  \]

- Note that \(B\) must be the same as \(C \rightarrow D\). We say that \(B\) must *unify* with \(C \rightarrow D\).

- The return type of `flip` is whatever the type of \(f \ x \ y\) is. After renaming the types, we have the type given at the top.
The type system is defined in terms of inference rules. For example, here is the rule for variables:

\[
(x : \tau) \in E \\
\frac{}{E \vdash x : \tau}
\]

and the one for function calls:

\[
E \vdash e_1 : \tau' \rightarrow \tau \quad E \vdash e_2 : \tau' \\
\frac{}{E \vdash e_1 \ e_2 : \tau}
\]

and here is the rule for if expressions:

\[
E \vdash e : \text{bool} \quad E \vdash e_1 : \tau \quad E \vdash e_2 : \tau \\
\frac{}{E \vdash \text{if} \ e \ \text{then} \ e_1 \ \text{else} \ e_2 : \tau}
\]
- fun exists pred [] = false
  | exists pred (x::xs) = pred x orelse
    exists pred xs;

val exists = fn : ('a -> bool) -> 'a list -> bool

- pred is a predicate: a function that returns a boolean
- exists checks whether pred is true for any member of the list

- exists (fn i => i = 1) [2, 3, 4];
  val it = false : bool
Applying functionals

- `exists (fn i => i = 1) [2, 3, 4];`
  `val it = false : bool`

Now partially apply `exists`:

- `val hasOne = exists (fn i => i = 1);`
  `val hasOne = fn : int list -> bool`
- `hasOne [3,2,1];`
  `val it = true : bool`
fun all pred [ ] = true
  | all pred (x::xs) = pred x andalso all pred xs

fun filter pred [ ] = [ ]
  | filter pred (x::xs) = if pred x
          then x :: filter pred xs
          else filter pred xs

all : (α → bool) → α list → bool

filter : (α → bool) → α list → α list
Let provides local scope:

(* standard Newton-Raphson *)

fun findroot (a, x, acc) =
  let val nextx = (a / x + x) / 2.0
     (* nextx is the next approximation *)
  in
    if abs (x - nextx) < acc * x
    then nextx
    else findroot (a, nextx, acc)
  end
A classic in functional form: mergesort

fun mrgSort op< [] = []
| mrgSort op< [x] = [x]
| mrgSort op< (a::bs) =
  let fun partition (left, right, []) =
    (left, right) (* done partitioning *)
  | partition (left, right, x::xs) =
    (* put x to left or right *)
    if x < a
    then partition (x::left, right, xs)
    else partition (left, x::right, xs)
  val (left, right) = partition ([], [a], bs)
  in
  mrgSort op< left @ mrgSort op< right
end

mrgSort : (α * α → bool) → α list → α list
Another variant of mergesort

```haskell
fun mrgSort op < [] = []
| mrgSort op < [x] = [x]
| mrgSort op < (a::bs) =
  let fun deposit (x, (left, right)) =
    if x < a
      then (x::left, right)
      else (left, x::right)
    val (left, right) = foldr deposit ([], [a]) bs
  in
  mrgSort op < left @ mrgSort op < right
end

mrgSort : (α * α → bool) → α list → α list
```
primitive types: `bool`, `int`, `char`, `real`, `string`, `unit`

constructors: `list`, `array`, `product` (tuple), `function`, `record`

“datatypes”: a way to make new types

structural equivalence (except for datatypes)

◆ as opposed to name equivalence in e.g. Ada

an expression has a corresponding type expression

the interpreter builds the type expression for each input

type checking requires that type of functions’ parameters match the type of their arguments, and that the type of the context matches the type of the function’s result
Records in ML obey structural equivalence (unlike records in many other languages).

A type declaration: *only needed if you want to refer to this type by name*

```
type vec = { x : real, y : real }
```

A variable declaration:

```
val v = { x = 2.3, y = 4.1 }
```

Field selection:

```
#x v
```

Pattern matching in a function:

```
fun dist {x,y} = 
sqrt ( pow (x, 2.0) + pow (y, 2.0))
```
A **datatype** declaration:

- defines a new type *that is not equivalent to any other type* (like name equivalence)
- introduces *data constructors*
  - *data constructors* can be used in patterns
  - they are also values themselves
datatype tree = Leaf of int  
  | Node of tree * tree  

Leaf and Node are data constructors:  
- Leaf : int → tree  
- Node : tree * tree → tree  

We can define functions by pattern matching:  

fun sum (Leaf t) = t  
  | sum (Node (t1, t2)) = sum t1 + sum t2
fun flatten (Leaf t) = [t] 
  | flatten (Node (t1, t2)) = 
      flatten t1 @ flatten t2

flatten : tree → int list

datatype 'a gentree =
  Leaf of 'a
  | Node of 'a gentree * 'a gentree

define names = Node (Leaf "this", Leaf "that")

names : string gentree
Pattern elements:

- integer literals: 4, 19
- character literals: \#'a\'
- string literals: "hello"
- data constructors: Node (⋯)
  - depending on type, may have arguments, which would also be patterns
- variables: x, ys
- wildcard: _

Convention is to capitalize data constructors, and start variables with lower-case.
More rules of pattern matching

Special forms:

- (), {} – the unit value
- [] – empty list
- [p1, p2, ⋯, pn] means (p1 :: (p2 :: ⋯ (pn :: [])⋯))
- (p1, p2, ⋯, pn) – a tuple
- {field1, field2, ⋯ fieldn} – a record
- {field1, field2, ⋯ fieldn, ⋯} – a partially specified record
- v as p – v is a name for the entire pattern p
option is a built-in datatype:

```plaintext
datatype 'a option = NONE | SOME of 'a
```

Defining a simple lookup function:

```plaintext
fun lookup eq key [] = NONE
    | lookup eq key ((k,v)::kvs) = 
      if eq (key, k) 
      then SOME v 
      else lookup eq key kvs
```

Is the type of `lookup`:

```plaintext
(\alpha \times \alpha \to \text{bool}) \to \alpha \to (\alpha \times \beta) \text{list} \to \beta \text{option}?
```

No! It's slightly more general:

```plaintext
(\alpha_1 \times \alpha_2 \to \text{bool}) \to \alpha_1 \to (\alpha_2 \times \beta) \text{list} \to \beta \text{option}
```
Another lookup function

We don’t need to pass two arguments when one will do:

```haskell
fun lookup _ [] = NONE
| lookup checkKey ((k,v)::kvs) = 
  if checkKey k
  then SOME v
  else lookup checkKey kvs
```

The type of this lookup:

```
(α → bool) → (α × β) list → β option
```
Useful library functions

- **map**: \( (\alpha \rightarrow \beta) \rightarrow \alpha \text{ list} \rightarrow \beta \text{ list} \)
  
  \[
  \text{map } (\text{fn } i \Rightarrow i + 1) \ [7, \ 15, \ 3] \\
  \Rightarrow [8, \ 16, \ 4]
  \]

- **foldl**: \( (\alpha \times \beta \rightarrow \beta) \rightarrow \beta \rightarrow \alpha \text{ list} \rightarrow \beta \)
  
  \[
  \text{foldl } (\text{fn } (a,b) \Rightarrow "(^a^+^b")") \ "0" \ ["1", \ "2", \ "3"] \\
  \Rightarrow "(3+(2+(1+0)))"
  \]

- **foldr**: \( (\alpha \times \beta \rightarrow \beta) \rightarrow \beta \rightarrow \alpha \text{ list} \rightarrow \beta \)
  
  \[
  \text{foldr } (\text{fn } (a,b) \Rightarrow "(^a^+^b")") \ "0" \ ["1", \ "2", \ "3"] \\
  \Rightarrow "(1+(2+(3+0)))"
  \]

- **filter**: \( (\alpha \rightarrow \text{bool}) \rightarrow \alpha \text{ list} \rightarrow \alpha \text{ list} \)
Overloading

Ad hoc overloading interferes with type inference:

```ml
fun plus x y = x + y
```

Operator `+` is overloaded, but types cannot be resolved from context (defaults to int). We can use explicit typing to select interpretation:

```ml
fun mix1 (x, y, z) = x * y + z : real
fun mix2 (x: real, y, z) = x * y + z
```
a function whose type expression has type variables applies to an infinite set of types

equality of type expressions means structural not name equivalence

all applications of a polymorphic function use the same body: no need to instantiate

```ml
let val ints = [1, 2, 3];
  val strs = ["this", "that"];
in
  len ints + (* int list -> int *)
  len strs   (* string list -> int *)
end;
```
An ML *signature* specifies an interface for a module.

```ml
signature STACKS =

sig

  type stack

  exception Underflow

  val empty : stack

  val push : char * stack -> stack

  val pop : stack -> char * stack

  val isEmpty : stack -> bool

end
```
structure Stacks : STACKS =

struct

  type stack = char list

  exception Underflow

  val empty = []

  val push = op:::

  fun pop (c::cs) = (c, cs)
  | pop [] = raise Underflow

  fun isEmpty [] = true
  | isEmpty _ = false

end