Assignment 5 Solutions

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Section 5.3

2 Counterexample:
Let B and C be any two disjoint sets (s.t. $B \cap C = \emptyset$). Then $A \cap (B \cap C)^c = A \cap U = A$.
Now any non-empty set A included in B will do it.
For a numerical example, let's take: $A = \{1\}, B = \{1,2\}, C = \{3,4\}$.

18 a The negation is: "$\exists S, \forall T$ s.t. $S \cap T \neq \emptyset$."
The statement is true. Take $T = \emptyset$.

b The negation is: "$\forall S, \exists T$ s.t. $S \cup T \neq \emptyset$."
The negation is true. Take $T = S^c$.

23 a Commutativity

b Distributivity

c Commutativity

27 We have:
$(A-B)-C = (A \cap B^c)-C = (A \cap B^c) \cap C^c = A \cap (B^c \cap C^c) = A \cap (B \cup C)^c = A-(B \cup C)$

Section 6.1

3 The sample space is:
$E = \{A\heartsuit, 2\heartsuit, 3\heartsuit, 4\heartsuit, 5\heartsuit, 6\heartsuit, 7\heartsuit, 8\heartsuit, 9\heartsuit, 10\heartsuit,$
$A\diamondsuit, 2\diamondsuit, 3\diamondsuit, 4\diamondsuit, 5\diamondsuit, 6\diamondsuit, 7\diamondsuit, 8\diamondsuit, 9\diamondsuit, 10\diamondsuit\}$
So the probability is $P(E) = \frac{20}{52} = .38$

4 The sample space is:
$E = \{2\clubsuit, 4\clubsuit, 6\clubsuit, 8\clubsuit, 10\clubsuit, 2\spadesuit, 4\spadesuit, 6\spadesuit, 8\spadesuit, 10\spadesuit\}$
So the probability is $P(E) = \frac{10}{52} = .19$

5 The sample space is:
$E = \{10\heartsuit, J\heartsuit, Q\heartsuit, K\heartsuit, A\heartsuit, 10\diamondsuit, J\diamondsuit, Q\diamondsuit, K\diamondsuit, A\diamondsuit,$
$10\clubsuit, J\clubsuit, Q\clubsuit, K\clubsuit, A\clubsuit, 10\spadesuit, J\spadesuit, Q\spadesuit, K\spadesuit, A\spadesuit, \}$
So the probability is $P(E) = \frac{20}{52} = .38$

6 The sample space is:
$E = \{2\clubsuit, 3\clubsuit, 4\clubsuit, 2\spadesuit, 3\spadesuit, 4\spadesuit, 2\heartsuit, 3\heartsuit, 4\heartsuit, 2\diamondsuit, 3\diamondsuit, 4\diamondsuit\}$
So the probability is $P(E) = \frac{12}{52} = .23$

11 a The sample space is:
$E = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$.

b i The event is: {HTT,THT,TTH}. The probability is $\frac{3}{8} = .38$.

ii The event is: {HHT,THH,HTH,HHH}. The probability is $\frac{4}{8} = .5$.

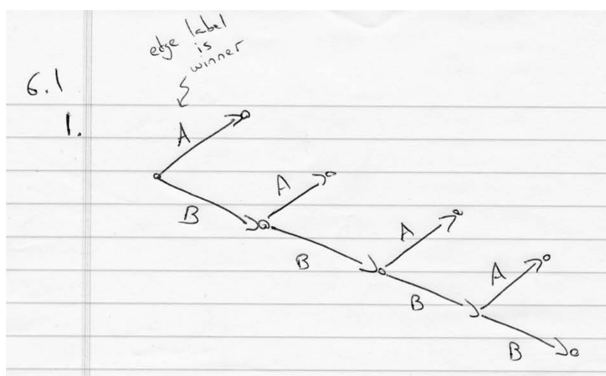
iii The event is: {TTT}. Its probability is $\frac{1}{8} = .13$

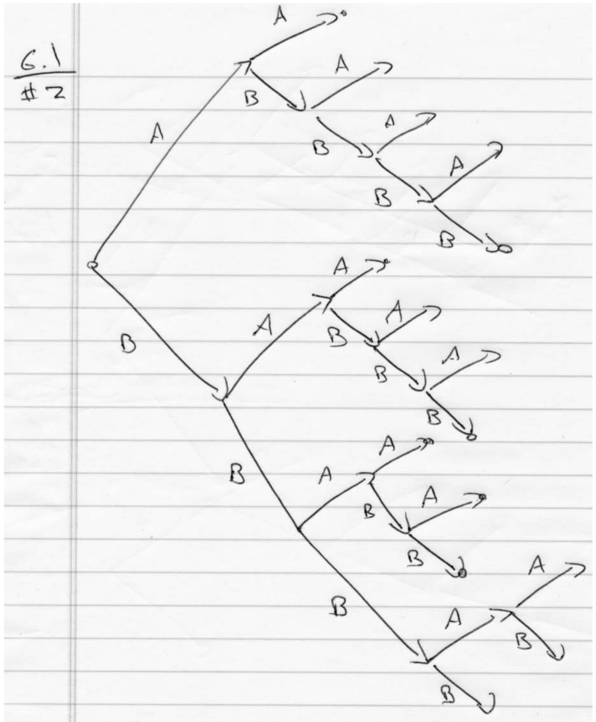
19 a The 25 possible outcomes are:
$\{B_1B_1, B_1B_2, B_1W_1, B_1W_2, B_1W_3,$
$B_2B_1, B_2B_2, B_2W_1, B_2W_2, B_2W_3,$
$W_1B_1, W_1B_2, W_1W_1, W_1W_2, W_1W_3,$
$W_2B_1, W_2B_2, W_2W_1, W_2W_2, W_2W_3,$
$W_3B_1, W_3B_2, W_3W_1, W_3W_2, W_3W_3\}$

b The event is:
$\{B_1B_1, B_1B_2, B_1W_1, B_1W_2, B_1W_3,$
$B_2B_1, B_2B_2, B_2W_1, B_2W_2, B_2W_3\}$.
Its probability is: $\frac{10}{25} = .4$

c The event is: $\{W_1W_1, W_1W_2, W_1W_3, W_2W_1, W_2W_2, W_2W_3, W_3W_1, W_3W_2, W_3W_3\}$
Its probability is: $\frac{9}{25} = .36$

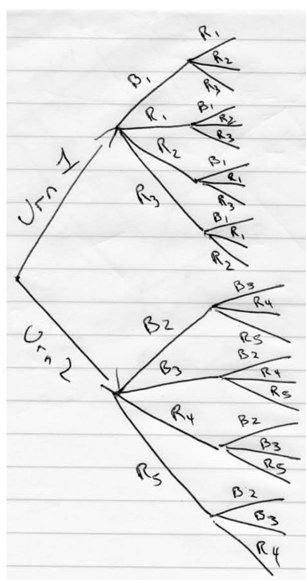
Section 6.2

1 

2 

3 There are four ways: $AAAA$, $BAAAA$,$BBAAA$, $BBBAAAA$.

4 There are two ways: $ABABABA$ and $BABABAB$.

7 a 

b 24

c $1/2 * (3/4 * 2/3) + 1/2(1/2 * 1/3) = 1/3$

10 a 12

b 12

14 a $26^4 * 10^3 = 456976000$

b $26^3 * 10^2 = 1757600$

c $10^3 = 1000$

d $26 * 25 * 24 * 23 * 10 * 9 * 8 = 258336000$

e $24 * 23 * 10 * 9 * 8 = 397440$

17 There are 14 groups. One way is to write it all out. The other way is to see that the total number of groups is $4 * 3 * 2$ with no restrictions. Then subtract the illegal combinations. The product rule won't work here because the number of choices are dependent on a previous step.

19 a 90

b 45

c $90 - 9 = 81$

d $45 - 5 = 40$

e $\frac{81}{90}$ and $\frac{40}{90}$ respectively

Section 6.3

2 a $16 + 16^2 + 16^3 = 4368$

b $16^2 + 16^3 + 16^4 + 16^5 = 1116464$

Bonus For the Tennis problem, a match of $6 - 4, 6 - 3, 1 - 6, 0 - 6, 7 - 6$ will do.