Theory of Computation
Sample Midterm.

This exam is closed book. Answer all questions.

1. (5 points). Give a DFA or an NFA to recognize the following language over the alphabet \(\{a, b\}\):

\[ A = \{w \mid w \text{ has at least two non-consecutive } b\text{'s}\}. \]

E.g. \(bbb \in A\), \(bbaa \notin A\), \(babaaab \in A\).

2. (5 points). Give a regular expression representing the following language over the alphabet \(\{a, b\}\).

\[ B = \{w \mid w \text{ does not end with } a\}. \]

Note that \(\lambda \in B\).

3. Consider the following DFA \(M\).

\[ \begin{array}{c}
    \hspace{1em} \buildrel a \over \longrightarrow \hspace{1em} \\
    p \quad \quad b, c \hspace{2em} \hspace{2em} b, c \hspace{1em} \buildrel a \over \longrightarrow \hspace{1em} q
    \end{array} \]

a. (6 points). Write descriptors \(D_p\) and \(D_q\) for the vertices \(p\) and \(q\), which specify those strings which cause \(M\) to end up at \(p\) and at \(q\), respectively. These descriptors can be in English, or regular expressions, or any other precise specification.

b. (4 points). Argue that for all strings \(x \in \{a, b\}^*\), if \(x \in D_p\), your descriptor for vertex \(p\), then \(xb \in D_q\), your descriptor for vertex \(q\).

4. a. (2 points). State the Pumping Lemma for regular languages.

b. (8 points). Show that the following language is not regular.

\[ E = \{a^ib^jc^k \mid i = j \text{ or } j = k \text{ or } i = k\}. \]

5. (10 points). Let \(\text{Subst-One-Char}(w, a, b) = \{x \mid w \text{ can be written as } w = uvv \text{ and } x = uvv\}\). This is the set of strings obtained by replacing an arbitrary one of the \(a\)'s in \(w\) by a \(b\).

E.g. \(\text{Subst-One-Char}(cacac, a, b) = \{cbcac, cacbc\}\).

Let \(\text{Subst-One-Char}(L, a, b) = \{x \mid x \in \text{Subst-One-Char}(w, a, b) \text{ for some } w \in L\}\).

Show that if \(L\) is regular, then so is \(\text{Subst-One-Char}(L, a, b)\). Remember to explain why your construction (a DFA or NFA) recognizes exactly \(L\).