1. In the article “The anatomy of a large-scale hypertextual web search engine” Brin and Page give their opinion of the effect of advertising on commercial search engines. What was this opinion? (One sentence only, please.)

2. Alice is planning to sell her newly developed product. She knows that both Ed and Bob are interested in buying the product, but she does not know how much they are willing to pay. Ed values the product at $130, and Bob values the product at $210 but these valuations are known only to themselves and not to Alice or the other buyer. Ed and Bob are trying to buy the product as cheaply as possible and certainly at or below their valuation for the product. Alice can run an auction between Ed and Bob to sell this product.

2.a. If Alice runs a second-price auction between Ed and Bob, who will end up getting the product and how much does he pay? What would Ed and Bob bid? Recall that a second price auction is a sealed bid auction (i.e. each bidder submits a bid in an envelope, say) and the winner is the highest bidder, and he or she pays the second highest bid. A tie, if any, is broken arbitrarily.

2.b. Answer the above question in the cases that Alice runs an English Auction and a Dutch Auction. What are the final prices of the product in each auction? Recall that in an English auction the bidders make successively higher bids increasing by a minimum increment of say $1, and the winner in the highest bidder and he or she pays the winning bid. While in a Dutch auction, the auctioneer calls out successively lower prices, starting at some high value, ideally above everyone’s valuation, decreasing by a decrement of say $1 from price to price, stopping when one bidder calls out “buy”; the item is then sold to that bidder at that price (ties are broken in an arbitrary manner).

2.c. Suppose that Alice runs a first-price auction between Bob and Ed. Assume that Ed is truthful and bids his value of $130 for the product and Bob knows Ed’s bid. What would Bob bid in this case? Is this auction truthful? Why? Recall that a first price auction is a sealed bid auction in which the highest bidder wins the item and pays their bid. Again, a tie, if any, is broken arbitrarily.

2.d. Assume that Alice runs the following probabilistic auction: She asks for Ed’s and Bob’s bid for the product, and suppose that their bids are $x$ and $y$ respectively. Then with probability $\frac{x}{x+y}$, she will give the item at price $x$ to Ed, and with probability $\frac{y}{x+y}$, she will give the product to Bob at price $y$. Is this auction truthful? Why or why not? Hint. i. What is the expected utility that Ed receives with a bid of $x$, assuming his value for the item is $v$? Recall that the utility is the value minus the price paid. The expectation here is over the randomness in Alice’s choice of winner. ii. Is bidding $x = v$ Ed’s best choice?

3. Consider the following example of the generalized second price auction: There are three advertisers competing in an auction for two sponsored search slots with bids $b_1 = $3, $b_2 = $10 and $b_3 = $4 and Click-through-rates $C_1 = 0.15$, $C_2 = 0.02$ and $C_3 = 0.01$ respectively.
3.a. If we run a generalized second price auction, which advertisers will end up in slots one and two respectively, and how much will each of them pay when there is a click on their assigned slot?

3.b. If the second advertiser’s click-through-rate, $C_2$, were equal to 0.03 (instead of 0.02), but everything else is unchanged, what changes are made in the allocation and payments? What if $C_2 = 0.08$?

3.c. In general, fixing everything else, if the clickthrough rate of an advertiser increases, what changes can happen to the payment or the allocation of the same advertiser? Support your claims by the above examples or new examples. Recall that each slot $i$ has an associated probability $\alpha_i$, where $\alpha_1 > \alpha_2$, and the probability of a click on slot $i$ filled with advertizer $j$’s ad is $\alpha_i \cdot C_j$.

4. Is the generalized second price auction used in the sponsored search auction a truthful auction? If yes, prove it. If not, show that it is not truthful through a counter-example to its truthfulness.