**Numerical Computing Programming Assignment 6**

This is a Matlab programming assignment about interpolation. It will involve modifying the interpolation routines demonstrated in class and now on the course web site. In addition to modifications needed to do the examples listed below, you should rewrite the necessary routines to use the Newton form of the interpolating polynomial, instead of the monomial basis and Vandermonde matrix approach demonstrated in class.

We will approximate a function using an $n-1$ degree interpolating polynomial at $n$ points, and then compute the largest error in the approximation. We will use different functions, different interpolating points, and different polynomial degrees, to get an idea of how they behave.

1. First, we will try using two different sets of points. First use equally spaced points in the interval [-1,1], (note that the Matlab routines use points in the interval [0,1] so you will have to change this too.) Second, use a set of points that are clustered at the endpoints, where we have seen the approximations behave the worst, defined by $t_k = cos(\frac{\pi}{2}\frac{(2k+1)}{n})$, $k = 0, ..., n-1$. What do these points look like?

2. Try different degree polynomials, using $n$ interpolating points, from $n = 3$ to $n = 10$. You can write a program that loops between 3 and 10 and plots the results. (Look at the Matlab "pause" command so you can look at the results before they are replaced by the next iterate.) Use the help utility in Matlab to learn how to add labels to the plots.

3. Apply your programs to the Runge function $f(x) = 1/(1 + 25x^2)$, as well as to the smoother function $f(x) = \exp(x^2)$. Feel free to experiment with other functions– for example, what happens if you try to interpolate a 5th degree polynomial by a 4th degree polynomial?

4. To compute the error in the approximate, write a routine that evaluates the function and the polynomial at 150 points, and takes the difference. Take the error to be the maximum of those. Make a table of the difference in error between the uniform points and the clustered points.