

Assignment 9. Graphs

Given April 18, due April 25.

1. We have a connected undirected graph, G , with all edges marked either red or blue (but not both). Let G_b be the subgraph having the same vertices but only the blue edges. The connected components of G_b are C_1, \dots, C_l . The “reduced graph” has one vertex for each of the C_k and an edge from C_k to C_j if there is a red edge in G that goes from a vertex in C_k to a vertex in C_j .
 - a. Suppose that C_k is connected to C_j in the above sense, and that $v \in C_k$, and $u \in C_j$ are two vertices of G . Show that there is a path in G from v to u that visits only vertices from C_k and C_j .
 - b. Show that, conversely, if there is a path from v to u as in part a., then the component containing v and the component containing u are connected in the reduced graph.
 - c. Sketch an algorithm that constructs the reduced graph (makes an array of vertices and edge lists for the vertices), using the set operations `makeset`, `find`, and `union`.
2. In a directed graph, the *indegree* of a vertex x is the number of edges whose head is x . The *outdegree* is the number of vertices whose tail is x . Suppose G is a directed graph given by an array of v vertices and, for each vertex $x \in V$, a linked list of other vertices in V . If y is in this linked list, then there is an edge whose tail is x and head is y . To find the outdegree of x we simply count the number of elements in the linked list at x . Give pseudocode that describes an algorithm that determines the indegree of each vertex. The work for this should be on the order of $e + v$.
3. For a directed graph described using the above data structure (a linked list of vertices to represent edges), give an algorithm related to breadth first search that creates a list, of all vertices that can be reached from x using no more than R hops. Call this list W . The work for this algorithm should be proportional to the number of vertices in W plus the number of edges whose tail is in W . *Warning:* This is a little subtle. The first step of normal breadth first search is to mark each vertex in V as “white” (CLR terminology) or “unvisited” (more common term). If V is much larger than W , this is not allowed. You must find a different way to determine whether a given vertex has been visited or not.