Assignment 1. **Order of magnitude and recurrence relations.**

Given January 22, due January 29.

1. For which of the following functions is it true that $f(2n) = O(f(n))$ as $n \to \infty$?

a. $f(n) = n$.

b. $f(n) = n^4$.

c. $f(n) = 2^n$.

d. $f(n) = \log(n)$.

e. $f(n) = n\log(n)$.

2. A certain algorithm works collections triangles. After n passes of the algorithm, the are T "active" triangles and P "passive" ones. In a pass of the algorithm, each active triangle is turned into 3 active triangles and 7 new passive ones. All the old passive triangles remain unchanged. Let $f(n)$ be the number of triangles after n passes if we start with a single active triangle. Show that $f(n) = \Theta(3^n)$.

3. One of the ways to estimate a sum is to estimate the area under the bar graph. Express the sum

$$f(n) = \sum_{k=2}^{n} \frac{1}{k\ln(k)}$$

as the area under a bar graph. Find functions related to $u(x) = 1/(x\ln(x))$ whose graph is just inside and just outside this bar graph. Use this to find an integrals involving $u(x)$ that are less than and more than $f(n)$. Use the identity $\frac{d}{dx}\ln(\ln(x)) = \frac{1}{x\ln(x)}$ to work this integral. Use this to show that $f(n) = \Theta\left(\ln(\ln(n))\right)$.

4. A data compression algorithm takes time proportional to n^2 to compress a block of n bytes of data. The program compresses a large data set by breaking it into blocks of n bytes and compressing these blocks one after another. The program takes 30 minutes to compress a gigabyte (GB) of data in 512 byte blocks. Assuming that all the time is spent in the compression algorithm, how long would it take to compress the same data if the block size were increased to 1 KB?