Theory of Computation
Homework 11.

Due Date: Monday, November 17.

Chapter 4:
No. 2,4,9,10,12,15.
For problem 9 you may assume the algorithm given in the proof of Lemma 3.6.2.
Chapter 3: No. 24, but read this as “Repeat Question 22” and not “Repeat Question 1”. (If you already did Question 23 in the form “Repeat Question 22” there is no need to do this question.

Challenge problem: Chapter 4:
No. 11.

Sample solution to Problem 13, Chapter 4.

$A_H(\langle P, w \rangle)$ computes as follows:

1. Compute $\langle R_{P,w} \rangle$, an input for $A_{\text{Mixed}}$. ($R_{P,w}$ is defined below.)
2. Simulate $A_{\text{Mixed}}(\langle R_{P,w} \rangle)$.
3. Report the answer given in Step 2.

For Step 3 to be correct we need to design $R_{P,w}$ so that

$\langle R_{P,w} \rangle \in \text{Mixed} \iff \langle P, w \rangle \in H. \quad (1)$

In other words:

$R_{P,w}$ halts on input 1 and does not halt on input 2 $\iff P(w)$ halts. \quad (2)

Consider the following candidate for program $R_{P,w}$:

$R_{P,w}(x)$:

\begin{verbatim}
if $x = 1$ then simulate $P(w)$
else loop forever
\end{verbatim}

$R_{P,w}$ never halts on input 2; it halts on input 1 exactly if $P(w)$ halts. Thus $R_{P,w}$ meets the requirement in Equation 2, which is also the requirement in Equation 1, and consequently the above algorithm $A_H$ decides $H$.\"