Theory of Computation
Sample Midterm.
This exam is closed book. Answer all questions.

1. (5 points). Give a DFA or an NFA to recognize the following language over the alphabet \{a, b\}^*:
   \[ A = \{ w \mid w \text{ has at least two non-consecutive } b's \} \]
e.g. \( bbb \in A, bbbaa \notin A, babaaab \in A \).

2. (5 points). Give a regular expression representing the following language over the alphabet \{a, b\}.
   \[ B = \{ w \mid w \text{ does not end with } a \} \]

3. Consider the following DFA \( M \).

   \[ a \quad b, c \quad b, c \]
   \[ p \quad q \]

   a. (6 points). Write descriptors \( D_p \) and \( D_q \) for the vertices \( p \) and \( q \), which specify those strings which cause \( M \) to end up at \( p \) and at \( q \), respectively. These descriptors can be in English, or regular expressions, or any other precise specification.
   b. (4 points). Argue that for all strings \( x \in \{a, b\}^* \), if \( x \in D_p \), your descriptor for vertex \( p \), then \( xb \in D_q \), your descriptor for vertex \( q \).

4. a. (2 points). State the Pumping Lemma for regular languages.
   b. (8 points). Show that the following language is not regular.
   \[ E = \{ a^i b^j c^k \mid i = j \text{ or } j = k \text{ or } i = k \} \]

5. (10 points). Let \( \text{Subst-One-Char}(w, a, b) = \{ x \mid w \text{ can be written as } w = uav \text{ and } x = ubv \} \).
   This is the set of strings obtained by replacing an arbitrary one of the \( a \)'s in \( w \) by a \( b \).
e.g. \( \text{Subst-One-Char}(cacac, a, b) = \{ cbcac, cacbc \} \).
Let \( \text{Subst-One-Char}(L, a, b) = \{ x \mid x \in \text{Subst-One-Char}(w, a, b) \text{ for some } w \in L \} \).
   Show that if \( L \) is regular, then so is \( \text{Subst-One-Char}(L, a, b) \). Remember to explain why your construction (a DFA or NFA) recognizes exactly \( L \).