1. Let $L$ be a listing of programs, i.e. $L(i) = Q_i$ for some program $Q_i$, where $Q_i(i)$ halts for all $i$, and further if $Q_i(x)$ halts, its output, denoted $Q_i(x)$, is an integer. Prove that there is a program $D$ such that, for every input $x$, $D(x)$ halts with an integer output, and yet $D$ is not in the listing $L$.

Chapter 4, Nos. 12,15,16,18.

Sample solution to Problem 13, Chapter 4.

$A_H((P,w))$ computes as follows:

1. Compute $\langle R_{P,w} \rangle$, an input for $A_{\text{Mixed}}$. ($R_{P,w}$ is defined below.)
2. Simulate $A_{\text{Mixed}}(\langle R_{P,w} \rangle)$.
3. Report the answer given in Step 2.

For Step 3 to be correct we need to design $R_{P,w}$ so that

$$\langle R_{P,w} \rangle \in \text{Mixed} \iff (P,w) \in H.$$  \hfill (1)

Note that

$$A_{\text{Mixed}}(\langle R_{P,w} \rangle) = \begin{cases} 
\text{"Recognize"} & \text{if } R_{P,w} \text{ halts on input 1 and runs forever on input 2.} \\
\text{"Reject"} & \text{if } R_{P,w} \text{ either runs forever on input 1, or halts on input 2, or both.}
\end{cases}$$

Recall that

$$A_H((P,w)) = \begin{cases} 
\text{"Recognize"} & \text{if } P(w) \text{ eventually halts} \\
\text{"Reject"} & \text{if } P(w) \text{ runs forever}
\end{cases}$$

So Equation 1 amounts to:

$$R_{P,w} \text{ halts on input 1 and does not halt on input 2 } \iff P(w) \text{ halts.} \tag{2}$$

Consider the following candidate for program $R_{P,w}$:

$$R_{P,w}(x):$$

if $x = 1$ then simulate $P(w)$
else loop forever

$R_{P,w}$ never halts on input 2; it halts on input 1 exactly if $P(w)$ halts. Thus $R_{P,w}$ meets the requirement in Equation 2, which is also the requirement in Equation 1, and consequently the above algorithm $A_H$ decides $H$. 

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